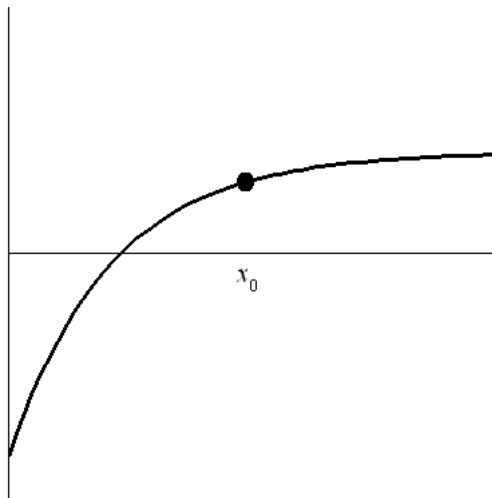


Math 4650 Sample Midterm

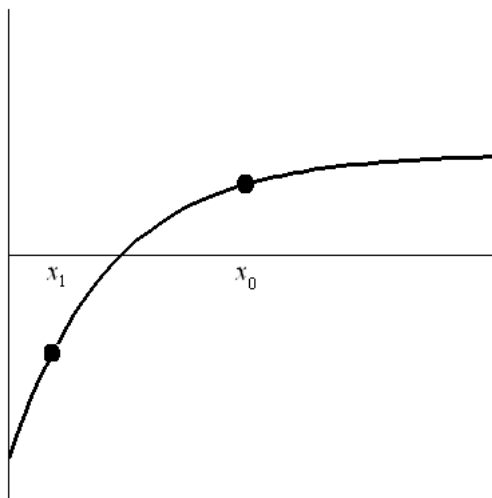
1. Derive, using Taylor series, the best possible algorithm for computing $f'(x_o)$ if you are given $f(x_o)$, $f(x_o + h)$, and $f(x_o + 3h)$. What is the first term in the error?
2. What does the following algorithm compute? Write the output for $N = 3$. (Don't simplify!) Then write a more efficient version of the algorithm.

```
Y=1
P=1
FOR K FROM 1 TO N DO
  P=1
  FOR J FROM 1 to K DO
    P=P*J
  END DO
  Y=Y+1/P
END DO
PRINT Y
```

3. If we run 8 steps of Newton's method on the following function, with initial condition as shown, will the estimate for the root be higher or lower than the actual root? Why?



If we run 12 steps of the secant method on the following function, with initial conditions as shown, will the estimate for the root be higher or lower than the actual root? Why?



4. A sequence is generated with $x_0 = 1$, $x_n = \sqrt{6 + x_{n-1}}$.
 - (a) What is the limit p of the sequence?
 - (b) What are the order of convergence and the asymptotic error constant?
5. Suppose you have values $\ln 1 = 0$, $\ln 2 = 0.693$, and $\ln 3 = 1.099$. Give a formula for an interpolation estimate of $\ln 2.5$ (don't multiply it out), and estimate how large the error could be.