

Math 4650 Homework #6 Solutions

1. 3.2 #7.

(a) Use Algorithm 3.2 to construct the interpolating polynomial of degree three for the unequally spaced points given in the following table:

x	$f(x)$
-0.1	5.30000
0.0	2.00000
0.2	3.19000
0.3	1.00000

The table of divided differences is

-0.1	5.30000			
0.0	2.00000	-33.00000		
0.2	3.19000	5.95000	129.83333	
0.3	1.00000	-21.90000	-92.83333	-556.66667

So the polynomial is

$$P_{0123}(x) = 5.30000 - 33.00000(x + 0.1) + 129.83333(x + 0.1)(x) - 556.66665(x + 0.1)(x)(x - 0.2).$$

(b) Add $f(0.35) = 0.97260$ to the table and construct the interpolating polynomial of degree four.

Adding the last point only changes the last row. The new divided differences are

$$0.35 \quad 0.97260 \quad -0.54800 \quad 142.34667 \quad 671.94286 \quad 2730.24340,$$

so the new interpolating polynomial is

$$P_{01234}(x) = 5.30000 - 33.00000(x + 0.1) + 129.83333(x + 0.1)(x) - 556.66665(x + 0.1)(x)(x - 0.2) + 2730.24340(x + 0.1)(x)(x - 0.2)(x - 0.3)$$

2. 3.3 #2a. Use Theorem 3.9 or Algorithm 3.3 to construct an approximating polynomial for the following data.

x	$f(x)$	$f'(x)$
0	1.00000	2.00000
0.5	2.71828	5.43656

We construct the divided difference table as in Table 3.13.

0	1.00000			
0	1.00000	2.00000		
0.5	2.71828	3.43656	2.87312	
0.5	2.71828	5.43656	4.00000	2.25376

Then we read off the coefficients from the diagonal:

$$P(x) = 1.00000 + 2.00000x + 2.87312x^2 + 2.25376x^2(x - 0.5)$$