

Math 4650 Homework #6 Solutions

1. Suppose you have a function  $g$  with known values  $g(2) = -1.225$ ,  $g(-1) = 0.518$ , and  $g(5) = 2.191$ . Estimate  $g'(1)$  by hand. Explain clearly how you are doing it.

**Solution:** Of course there are many ways to do this. The easiest is to construct the second-order Lagrange interpolating polynomial at the three points. We have

$$\begin{aligned} P(x) &= -1.225 \frac{(x+1)(x-5)}{(2+1)(2-5)} + 0.518 \frac{(x-2)(x-5)}{(-1-2)(-1-5)} + 2.191 \frac{(x+1)(x-2)}{(5+1)(5-2)} \\ &= 0.136(x^2 - 4x - 5) + 0.0288(x^2 - 7x + 10) + 0.122(x^2 - x - 2). \end{aligned}$$

Therefore

$$P'(x) = 0.136(2x - 4) + 0.0288(2x - 7) + 0.122(2x - 1) = 0.574x - 0.868,$$

so that  $g'(1) \approx P'(1) = -0.294$ .