

Math 4650 Midterm

Name: _____

Instructions:

- Do any **five** of the problems. If you try more than five, you must clearly label which five you want graded.
- Attach your formula sheet to the exam when finished.
- Show work when feasible. When using results from class, cite them clearly.
- Don't cheat.
- Stay calm, confident, and calculating correctly.

I would like the following problems to be graded (circle five):

1 2 3 4 5 6 7

(For instructor use only:)

1	/20
2	/20
3	/20
4	/20
5	/20
6	/20
7	/20
Total	/100

1. Regula Falsi (false position) was proposed as a method that combines the best of two methods: bisection and the secant method.

Explain, using words, diagrams, and/or formulas, why it's actually worse than either method in practice. (There are several correct answers.)

2. Consider the sequence $p_n = \frac{1}{n-1}$ for $n \geq 2$.

(a) Is p_n linearly convergent? If so, what is the asymptotic error constant?

(b) Use Aitken's method to get a new sequence \hat{p}_n , and fully simplify the formula for it. Does the new sequence converge substantially faster than the old one?

3. (a) Estimate $\int_0^1 x^3 dx$ using the trapezoid rule with $n = 1$. What is the actual error?

(b) What is the maximum error predicted by the general error formula?

4. Knowing that $f(0) = 2$, $f'(0) = 8$, $f(2) = 6$, and $f'(2) = 0$, estimate $f(1)$.

5. Consider the following algorithm.

```
X = 1
Err = 1
WHILE ERR>0.0001 DO
  Xnew = 3X - 1/X
  Err = ABS(Xnew-X)
  X = Xnew
END DO
PRINT X
```

(a) Go through the loop three times. What X comes out?

(b) How many steps will this program take?

(c) What X do you think the programmer was trying to approximate?

(d) Why would this program not work as expected?

6. Suppose you estimate $f'(3)$ using the formula from class,

$$f'(3) \approx \frac{4f(3+h) - f(3+2h) - 3f(3)}{2h}.$$

When $h = 0.2$, you obtain $f'(3) \approx 4.8$.

When $h = 0.1$, you obtain $f'(3) \approx 4.5$

Use Richardson's extrapolation to obtain the best estimate of the true value of $f'(3)$.

7. While driving from Boulder to Denver on Route 36 during rush hour to measure traffic congestion, a traffic engineer recorded the car's speed from the speedometer every five minutes and got the following values.

time (min)	0	5	10	15	20	25	30	35	40	45	50	55	60
speed (mph)	0	23	57	65	28	17	45	71	64	44	22	31	0

- (a) If you were trying to determine the exact time at which the speed was lowest, what method would you use and why? (Don't actually *do* it, just describe the process in enough detail that anyone could program it and get the same answer.)
- (b) If you were trying to determine where on Route 36 traffic was the worst, how would you get that position from this data? Why would you use that method? (Again, don't do it, just describe the process precisely.)