Review Sheet for Second Exam

Mathematics 2300

October 18, 2006

The exam will cover Sections 7.7, 7.9, 8.2, 8.3, 8.4, 8.5, 8.7, 8.8, and 10.1.
No calculators of any kind will be allowed.

Formulas to remember:

- Work performed on a particle by a force $F(x)$:

$$W = \int_a^b F(x)\,dx$$

- Work performed lifting a large object:

  $W = \sum$ (vertical distance each horizontal slice travels)$\times$(weight of each horizontal slice)

  (Take the limit as the thickness of each slice goes to zero, to get an integral.)

- The midpoint sum with n intervals for $\int_a^b f(x)\,dx$ is

$$M_n = \big[f(x_{1/2}) + f(x_{3/2}) + f(x_{5/2}) + \cdots + f(x_{n-1/2})\big]\Delta x$$

  where $\Delta x = \frac{b-a}{n}$ and $x_{k+1/2} = a + (k+1/2)\Delta x$.

- The trapezoid sum with n intervals is

$$T_n = \big[\frac{1}{2}f(x_0) + f(x_1) + f(x_2) + \cdots + f(x_{n-1}) + \frac{1}{2}f(x_n)\big]\Delta x$$

  where $x_k = a + k\Delta x$.

- Simpson's Rule with $2n$ intervals is

$$S_{2n} = \frac{2M_n + T_n}{3}.$$

  The number of midpoints vs. trapezoids can be remembered using the Simpsons-based mnemonic "Meet Troy McClure."

- Improper integrals: if $f(x)$ has no singularities in $[a, b)$ and a vertical asymptote at $x = b$ (or $b = +\infty$), then

$$\int_a^b f(x)\,dx = \lim_{k \to b^-} \int_a^k f(x)\,dx.$$

- If $f(x)$ has no singularities in $(a, b]$ and a vertical asymptote at $x = a$ (or $a = -\infty$), then

$$\int_a^b f(x)\,dx = \lim_{k \to a^+} \int_k^b f(x)\,dx.$$

- $L = \lim_{n \to \infty} a_n$ if, for every small positive number ε, we can find an integer N such that $|a_n - L| < \varepsilon$ for every $n > N$.

- If a sequence is defined recursively by a formula $a_{n+1} = f(a_n)$ with f continuous, and if a_n is known to have a limit L, then $L = f(L)$.

Be able to:

- Set up the geometry of a work problem, when some object is lifted (for example, water being lifted out of a container). Understand how to obtain an integral from this.

- Draw rectangles representing the left sum and right sum approximations, and draw trapezoids representing the midpoint sum and trapezoid sum approximations for a given function. (Remember one can always draw the midpoint sum as the area under a tangent line to the curve at the midpoint.) Understand how these pictures tell you whether particular sums are larger or smaller than the actual area (using the function's direction for left/right sums and the function's concavity for midpoint/trapezoid sums).

- Identify improprieties in integrals (either vertical asymptotes within the interval, or limits at $-\infty$ or $+\infty$). Break up an improper integral into integrals of the same function over smaller intervals, such that each integral has only one impropriety, and the impropriety occurs at the endpoint. Understand that the integral converges when *every* one of the smaller-interval integrals converges, and diverges if *any* of the smaller-interval integrals diverges.

- Compute limits of improper integrals, using techniques such as L'Hopital's Rule (review this if you forget). Remember that $\frac{0}{0}$ or $\infty - \infty$ are very common results when doing improper integrals, and that these do *not* imply divergence. They imply you have to do more algebra.

- Prove that a sequence given by a formula $a_n = g(n)$ converges to some limit, using the definition. (For example, to prove $a_n = 1/n^2$ converges to 0, you have to choose $N = 1/\sqrt{\varepsilon}$; then whenever $n > N$, we will have $|a_n - 0| < \varepsilon$.)

- Write the first few terms of a sequence defined by a recursive formula. Also be able to compute the limit, if one exists, of a sequence defined recursively.