

Math 4001 Analysis 2
Homework Set 6

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Problem 1: Consider the map

$$f : \mathbb{R}^n \setminus \{0\} \rightarrow \mathbb{R}^n, \quad x \mapsto \frac{x}{x_1^2 + \dots + x_n^2}$$

is differentiable at each point $x \in \mathbb{R}^n \setminus \{0\}$, and determine its derivative by computing the Jacobian at each point. (6P)

Problem 2: Let

$$g : \mathbb{R}^2 \rightarrow \mathbb{R}, \quad (x, y) \mapsto \begin{cases} 0, & \text{if } (x, y) = (0, 0), \\ \frac{x^3}{x^2+y^2} & \text{if } (x, y) \neq (0, 0). \end{cases}$$

Prove that g is everywhere partial differentiable on its domain. Determine all partial derivatives. Where is g differentiable? (6P)

Problem 3: Let $f, g : \Omega \rightarrow \mathbb{R}$ be two differentiable functions, where $\Omega \subset \mathbb{R}^n$ is open. Prove that the product $f \cdot g$ is differentiable, and determine a formula for the Jacobian of $f \cdot g$. (4P)