Math 4001 Analysis 2
Homework Set 5

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Problem 1: For which $z \in \mathbb{C}$ do the following limits exist:

$$\text{a)} \lim_{n\to\infty} z^n, \qquad \text{b)} \lim_{n\to\infty} n!\, z^n .$$

(6P)

Problem 2: Let ϱ_1 be the convergence radius of $\sum_{k=0}^{\infty} a_k\, z^k$ and ϱ_2 the convergence radius of $\sum_{k=0}^{\infty} b_k\, z^k$. Prove the following propositions:
a) If $|a_k| \leq |b_k|$ for all $k \in \mathbb{N}$, then $\varrho_1 \geq \varrho_2$.
b) The convergence radius of $\sum_{k=0}^{\infty}(a_k + b_k)\, z^k$ is $\geq \min(\varrho_1, \varrho_2)$.

(6P)

Problem 3: Let $f_k(z) = \frac{1}{1+az^k}$ with $a \neq 0$. Prove that $(f_k)_{k\in\mathbb{N}}$ converges uniformly to 1 on $B_r(0)$ for $0 \leq r < 1$, and uniformly to 0 on $\mathbb{C} \setminus B_R(0)$ for $R > 1$. Does $(f_k)_{k\in\mathbb{N}}$ convergence on $\mathbb{C}$? (6P)

Problem 4: Determine the Taylor series of the function $\sqrt{1+x}$, $-1 < x < 1$ around $x_0 = 0$, its convergence radius, and check whether $\sqrt{1+x}$ is analytic. Hint: You can use the binomial series. (6P)