

**Math 4001 Analysis 2**  
**Homework Set 3**

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**Problem 1:** Let  $D \subset \mathbb{C}$ . Prove that the product of two functions  $f, g : D \rightarrow \mathbb{C}$  which are both continuous at  $z_0 \in D$  is continuous at  $z_0$  as well. (4P)

**Problem 2:** Prove that the geometric series  $\sum_{k=0}^{\infty} z^k$  does not converge uniformly on the unit circle  $B_1(0) := \{z \in \mathbb{C} \mid |z| < 1\}$ . (4P)

**Problem 3:** Show that the following sequences of functions converge pointwise on  $\mathbb{R}_{\geq 0}$ . Determine the limits. Determine in both cases whether the function sequence converges uniformly.

$$a) \quad f_n(x) = \frac{1}{1+nx} \quad b) \quad f_n(x) = \frac{x}{1+nx}. \quad (4P)$$

**Problem 4:** Compute the supremum norms of the functions  $f_n : \mathbb{R} \rightarrow \mathbb{R}, x \mapsto \frac{|x|^n}{1+|x|^n}, n \in \mathbb{N}$ . Determine the convergence properties of the function sequence  $(f_n)_{n \in \mathbb{N}}$ . (4P)