

Math 4001 Analysis 2
Homework Set 2

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Problem 1: a) Show that the following series converges and compute its sum:

$$\sum_{n=1}^{\infty} \frac{1}{4n^2 - 1}$$

b) For which $z \in \mathbb{C}$ does the following series converge:

$$\sum_{n=1}^{\infty} \frac{z^{2n}}{(1 + |z|^2)^{n-1}}$$

Compute its sum in the converging case. (6P)

Problem 2: Let a and b be two complex numbers. Define the sequence $(a_n)_{n \in \mathbb{N}}$ recursively as follows:

$$a_0 := a, \quad a_1 := b, \quad a_{n+2} := \frac{1}{2}(a_{n+1} + a_n).$$

Prove that the sequence $(a_n)_{n \in \mathbb{N}}$ converges and determine its limit. Proceed as follows. First find a formula for the difference $a_{n+1} - a_n$ in terms of $a_1 - a_0$. Use this formula to write the difference $a_n - a_m$ for $n > m$ as a (finite) geometric series. From the latter conclude that $(a_n)_{n \in \mathbb{N}}$ is a Cauchy sequence and determine its limit. (6P)

Problem 3: Prove that for all $z \in \mathbb{C}$

$$\cosh z = \sum_{n=0}^{\infty} \frac{1}{(2n)!} z^{2n} \quad \text{and} \quad \sinh z = \sum_{n=0}^{\infty} \frac{1}{(2n+1)!} z^{2n+1}.$$

(4P)