Math 6220 Introduction to Topology 2
Homework Set 6

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Problem 1: Prove that the infinite cylinder $R \times S^1$ is a covering space for the torus.

Problem 2: Let p, q be two relatively prime integers. Consider the 3-sphere

$$S^3 := \big\{ (z_0, z_1) \in \mathbb{C}^2 \mid |z_0|^2 + |z_1|^2 = 1 \big\}.$$

Let $\zeta = e^{\frac{2\pi i}{p}}$ be a primitive p-th root of unity, and consider the action $\mathbb{Z}/p\mathbb{Z} \times S^3 \to S^3$,

$$\big(l + p\mathbb{Z}, (z_0, z_1)\big) \mapsto \big(\zeta^l z_0, \zeta^{lq} z_1\big), \quad l = 0, \ldots, p-1.$$

Denote by $L(p,q)$ the quotient space of this group action (with its inherited topology) and call it a lens space.

a) Show that $L(p,q)$ is a compact Hausdorff space.

b) Prove that $L(1,1) = S^3$ and $L(2,1) = \mathbb{RP}^3$.

c) Prove that $L(p,q) = L(p,q')$, if $q \equiv q' \mod p$.

d) Show that $S^3 \to L(p,q)$ is a covering and determine its number of sheets.

e) Determine the fundamental group of the lens space $L(p,q)$.