Math 6220 Introduction to Topology 2
Homework Set 5

Spring 2008

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Problem 1: Prove that the functor which associates to each presheaf $\mathcal{F}$ on a topological space X the sheafification $\hat{\mathcal{F}}$ satisfies the following universal property: For each sheaf $\mathcal{G}$ on X and each presheaf morphism $\eta : \mathcal{F} \to \mathcal{G}$ there exists a unique sheaf morphism $\hat{\eta} : \hat{\mathcal{F}} \to \mathcal{G}$ such that $\eta = \hat{\eta} \circ \iota$ with $\iota : \mathcal{F} \to \hat{\mathcal{F}}$.

Problem 2: Let X and Y be Hausdorff topological spaces, and $A \subset X$ a compact subspace. Additionally, let $f : A \to Y$ be a continuous map. Show that then $X \coprod_f Y$ is Hausdorff.

Problem 3: Show that

$$\mathbb{CP}^n \cong e^0 \cup e^2 \cup \ldots \cup e^{2n}$$

and determine from this the homology and the Euler characteristic of the complex projective spaces.