

**Math 6220 Introduction to Topology 2**  
**Homework Set 1**

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**Problem 1:** Show that the Stone-Čech compactification is a functor from the category of completely regular topological spaces to the category of compact topological spaces.

**Problem 2:** a) (First isomorphism theorem in **Comp**, the category of chain complexes of a given ring  $R$ ) Let  $f : C_\bullet \rightarrow D_\bullet$  be a chain map between chain complexes. Show that there is a (uniquely determined) isomorphism

$$\theta : C_\bullet / \ker_\bullet f \cong \operatorname{im}_\bullet f,$$

such that  $\iota \circ \theta \circ \pi = f$ , where  $\iota : \operatorname{im}_\bullet f \hookrightarrow D_\bullet$  is the canonical injection and  $\pi : C_\bullet \rightarrow C_\bullet / \ker_\bullet f$  is the canonical projection.

b) (Second isomorphism theorem in **Comp**) If  $C_\bullet$  and  $D_\bullet$  are subcomplexes of  $S_\bullet$ , prove that the following isomorphism holds in **Comp**:

$$C_\bullet / (C_\bullet \cap D_\bullet) \cong (C_\bullet + D_\bullet) / D_\bullet.$$

**Problem 3:** Let  $C_\bullet^\lambda$ ,  $\lambda \in \Lambda$  be a family of chain complexes. Show that for each  $n \in \mathbb{N}$

$$H_n \left( \bigoplus_{\lambda \in \Lambda} C_\bullet^\lambda \right) \cong \bigoplus_{\lambda \in \Lambda} H_n(C_\bullet^\lambda).$$