

Math 4310 Introduction to Analysis
Final Exam

Spring 2008

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Problem 1: Find the value of the following series:

$$\sum_{k=0}^{\infty} \frac{1}{k^2 + 3k + 2},$$

(4P)

Problem 2 a: For $k \in \mathbb{N}^*$ determine the limits

$$\lim_{t \rightarrow 0} \frac{1}{t^k} \exp\left(-\frac{1}{t^2}\right)$$

(4P)

Problem 2 b: Prove that the function

$$f : \mathbb{R} \rightarrow \mathbb{R}, x \mapsto \begin{cases} \exp\left(-\frac{1}{x^2}\right) & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$$

is \mathcal{C}^∞ , and determine all derivatives $f^{(k)}(0)$, $k \in \mathbb{N}$. (6P)

Problem 3: Prove the following relation, where n denotes a positive integer:

$$\int \cos^n x \, dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x \, dx.$$

(6P)

Problem 4: Determine the following integrals:

$$\text{a) } \int \frac{\ln x}{x^2} \, dx \quad \text{b) } \int \cosh^3 x \, dx$$

(6P)

Problem 5: Let $f : [a, b] \rightarrow [a, b]$ be continuous. Show that then f has a fixed point, which means that there is $x \in [a, b]$ with $f(x) = x$. (6P)