

Math 4310 Introduction to Analysis
Homework Set 4

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Problem 1: Check which of the following sequences converges, and determine its limit, if it does:

$$\begin{array}{ll} \text{a)} & x_n = \frac{1+(-1)^n}{n} \\ \text{c)} & x_n = \frac{n^2}{n^2+1} \end{array} \qquad \begin{array}{ll} \text{b)} & x_n = \frac{n^3}{n^2+1} \\ \text{d)} & x_n = \frac{n^2-n}{n^3+1} \end{array} \qquad (4P)$$

Problem 2: Prove that the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ converges. Deduce from this that for each natural

$k \geq 2$ the series $\sum_{n=1}^{\infty} \frac{1}{n^k}$ converges. (6P)

Problem 3: Let $(a_n)_{n \in \mathbb{N}}$ be a sequence of real numbers such that $a_n \geq r$ for some $r > 0$. Show the following:

- a) The series $\sum a_n$ diverges.
- b) The series $\sum \frac{a_n}{1+a_n}$ diverges.
- c) The series $\sum \frac{a_n}{1+n^2 a_n}$ converges.

(6P)