

Math 4310 Introduction to Analysis
Homework Set 3

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Problem 1: Show that the sequences $\left(\frac{n}{n+1}\right)_{n \in \mathbb{N}}$ and $\left(\frac{n}{2^n}\right)_{n \in \mathbb{N}}$ converge and determine their limits. (4P)

Problem 2: Prove by induction that for each $x \in \mathbb{R} \setminus \{1\}$

$$\sum_{k=0}^n x^k = \frac{1 - x^{n+1}}{1 - x}.$$

Does the limit $\lim_{n \rightarrow \infty} \sum_{k=0}^n x^k$ exist? (4P)

Problem 3: Using the binomial formula show that for real $x \geq 0$ and every natural number $n \geq 2$ one has

$$(1 + x)^n \geq \frac{n^2}{4} x^2. \tag{4P}$$

Problem 4: Prove that for real $a, b \geq 0$ one has

$$\sqrt{ab} \leq \frac{1}{2}(a + b). \tag{4P}$$