

**MATH 6230 Differential Geometry 1**  
**Homework Set 6**

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**Problem 1:** Let  $\alpha$  be the following 1-form on  $\mathbb{R}^2 \setminus \{0\}$ :

$$\alpha = \frac{1}{2\pi} \frac{xdy - ydx}{x^2 + y^2}.$$

Prove that  $d\alpha = 0$ , and compute the integral

$$\int_{S^2} \alpha.$$

Is there a smooth function  $f$  on  $\mathbb{R}^2 \setminus \{0\}$  with  $df = \alpha$  ?

**Problem 2:** Which of the real projective spaces  $\mathbb{P}^n$  is orientable, which is not?

**Problem 3:**

- a) Determine the coefficients of the standard metric on the  $n$ -sphere  $S^n$  in coordinates given by the stereographic projection.
- b) Determine the coefficients of the standard metric on the 2-torus  $T^2$  in appropriate coordinates.

**Problem 4:** Compute the Christoffel symbols of the Levi-Civita connection on  $S^n$  and  $T^2$  for the above coordinate systems.