MATH 6230 Differential Geometry 1
Homework Set 2

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Problem 1: Let V be a finite dimensional real or complex vector space, and $k \in \mathbb{N}^*$. Define the *Graßmannian* $\mathrm{Gr}_k(V)$ as the set of all k-dimensional subspaces of V. Define a manifold structure on $\mathrm{Gr}_k(V)$ which extends in a natural way the construction of a manifold structure on projective space.

Problem 2: Prove that the group of orthogonal $(n \times n)$-matrices $\mathrm{O}(n)$ is a submanifold of $\mathrm{GL}(n, \mathbb{R})$ of dimension $\frac{1}{2}n(n-1)$.

Problem 3: For positive n let $M := \{x \in \mathbb{R}^n \mid x_1^2 = x_2^2 + \ldots + x_n^2, \ \& \ x_1 \geq 0\}$. Prove that M is not a submanifold of $\mathbb{R}^n$.

Problem 4: Prove that the composition and the cartesian product of two embeddings is again an embedding.