Math 3001 Analysis 1
Homework Set 7

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Problem 1 Compute the derivatives of the following functions:

a) $f : \mathbb{R} \to \mathbb{R},\ x \mapsto \sin\left(e^x\right)$,

b) $g : \mathbb{R} \to \mathbb{R},\ x \mapsto x \ln\left(\frac{1}{1+x^2}\right)$,

c) $h : \mathbb{R}_{>0} \to \mathbb{R},\ x \mapsto x^x$.

(6P)

Problem 2: We call a subset of a topological space clopen if it is both open and closed. Show that there are no clopen subsets of the real line except the empty set and the entire line.
Hint: Remember that a set in the reals is open if it is the union of open intervals (even infinitely many) and closed if its compliment is open. (4P)

Problem 3: Use the previous result to prove the Intermediate Value Theorem.
Hint: Recall that a function f is continuous if and only if the preimage of an open set is open. (6P)

Problem 4: Determine the integral

$$\int e^{-x} \cos(5x)\, dx.$$

(4P)