Math 3001 Analysis 1
Homework Set 5

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Course Instructor: Dr. Markus Pflaum

Contact Info: Office: Math 255, Telephone: 2-7717, e-mail: `markus.pflaum@colorado.edu`.

Problem 1: Prove for $x, y > 0$ the following inequality:

$$|\sqrt{x} - \sqrt{y}| \leq \frac{|x-y|}{2\min\{\sqrt{x}, \sqrt{y}\}}.$$

Derive from this that the function $\mathbb{R}^+ \to \mathbb{R}$, $x \mapsto \sqrt{x}$ is continuous. (4P)

Problem 2: a) Show that for $x \in \mathbb{R}$ and all $n \in \mathbb{N}^*$

$$x^n - 1 = (x-1)\left(1 + x + \ldots + x^{n-1}\right)$$

(2P)

b) Let $n, m \in \mathbb{N}^*$, and consider the function

$$f : \mathbb{R} \setminus \{1, -1\} \to \mathbb{R}, \quad x \mapsto \frac{x^n - 1}{x^m - 1}.$$

Determine the limit $\lim_{x \to 1} f(x)$. (2P)

Problem 3: For $k \in \mathbb{N}^*$ determine the limits

$$\lim_{t \to 0} \frac{1}{t^k} \exp\left(-\frac{1}{t^2}\right)$$

(4P)

Problem 4: a) Determine for the following function whether it is continuous at the point $x_0 = 0$.

$$f : \mathbb{R} \to \mathbb{R}, \quad x \mapsto f(x) = \begin{cases} \frac{x-6}{x-3} & \text{if } x < 0 \\ 2 & \text{if } x = 0 \\ \sqrt{4 + x^2} & \text{if } x > 0 \end{cases}$$

(4P)

b) Define the maximal domain $D \subset \mathbb{R}$ on which

$$g : D \to \mathbb{R}, \quad x \mapsto g(x) = \frac{x^2 + 3x + 5}{x^2 + 3x - 4}$$

is a continuous function. (4P)