Math 2002 Number Systems
Homework Set 2

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Problem 1: Prove that $\big((p \implies q) \wedge (q \implies r)\big) \implies (p \implies r)$ is a tautology.

a) by using truth tables,

b) by using logic equivalence laws.

(4P)

Problem 2: Suppose you have predicates $A(x)$, $E(x)$, and $W(x)$. Negate the following logical statements and then push all negations inward so that they are only acting on the predicates $A(x)$, $E(x)$, and $W(x)$. Also, state whether the statement is a predicate or a proposition.

a) $\forall x\big(A(x) \implies E(x)\big)$

b) $\exists x\big(E(x) \wedge \neg W(x)\big)$

(4P)

Problem 3: Translate the following sentences into symbolic logic.

For every positive number ε, there is a positive number δ for which the relation $|x - a| < \delta$ implies $|f(x) - f(a)| < \varepsilon$.

(2P)

Problem 4: Show that the subset relation $\subset$ is transitive. (2P)

Problem 5: Consider the mathematical theory having equality $=$ and $\leq$ as its (non-logical) symbols with the following axioms (plus the axioms for equality from class):

a) (reflexivity) $\forall x(x \leq x)$.

b) (antisymmetry) $\forall x \forall y\big((x \leq y) \wedge (y \leq x)\big) \implies (x = y)$.

c) (transitivity) $\forall x \forall y \forall z\big((x \leq y) \wedge (y \leq z)\big) \implies (x \leq z)$.

Prove that the following statement is a theorem in that theory:

$$\forall M \forall N \forall x\big(x \leq M \wedge x \leq N\big) \implies (M = N)$$

Provide an argument whether the following statement can be derived from the axioms:

$$\exists M \forall x\big(x \leq M\big)$$

(4P)

Problem 6: Let M, N, L be sets.
a) Prove the following rule of de Morgan:

$$M \setminus (N \cup L) = (M \setminus N) \cap (M \setminus L).$$

b) Prove the following distributivity law:

$$M \cap (N \cup L) = (M \cap N) \cup (M \cap L).$$

(4P)