Solutions, exercise set 1

$\boxed{\textbf{Chapter 1,}\text{ exercise 1}}$ (a): We have $y \in \{x, y\} = \{x, z\}$, so $y = x$ or $y = z$. If $y = x$, then $z \in \{x, z\} = \{x, y\} = \{x\}$, so $z = x$; hence $y = x = z$. So $y = z$ in either case.

(b): We have $u \in \{x, u\} = \{y, v\}$, so $u = y$ since $u \neq v$. Similarly, $v = x$.

$\boxed{\textbf{Chapter 1,}\text{ exercise 3}}$ Suppose that $x \in a \triangle (b \triangle c)$. Then there are two possibilities.

Case 1. $x \in a$ and $x \notin b \triangle c$. The second condition implies that x is in both b and c or is in neither of them. So x is in just a, or is in all three.

Case 2. $x \notin a$ and $x \in b \triangle c$. The second condition implies that x is in exactly one of b, c. Hence x is in exactly one of a, b, c.

Conversely, suppose that x is in exactly one of a, b, c. If $x \in a$ and $x \notin b, c$, then $x \in a$ and $x \notin b \triangle c$, so $x \in a \triangle (b \triangle c)$. If $x \in b$ and $x \notin a, c$, then $x \in b \triangle c$ and $x \notin a$, so $x \in a \triangle (b \triangle c)$. If $x \in c$ and $x \notin a, b$, then $x \in b \triangle c$ and $x \notin a$, so $x \in a \triangle (b \triangle c)$.

If x is in all three of a, b, c, then $x \in a$ and $x \notin b \triangle c$, so $x \in a \triangle (b \triangle c)$.

$\boxed{\textbf{Chapter 1,}\text{ exercise 4}}$ For any set x,

$$\begin{aligned} x \in a \triangle (b \triangle c) \quad &\text{iff} \quad x \text{ is in just one, or all three, of } a, b, c \\ &\text{iff} \quad x \in c \triangle (a \triangle b) \\ &\text{iff} \quad x \in (a \triangle b) \triangle c. \end{aligned}$$

$\boxed{\textbf{Chapter 1,}\text{ exercise 13}}$ Let $x = a \cup (c \backslash b)$. Then

$$\begin{aligned} b \cap x &= b \cap (a \cup (c \backslash b) \\ &= (b \cap a) \cup (b \cap (c \backslash b)) \\ &= a \cup \emptyset = a \quad \text{and} \\ b \cup x &= b \cup a \cup (c \backslash b) \\ &= b \cup (c \backslash b) \\ &= c. \end{aligned}$$