

Errata 1

Pages 51, 52 Replace the part of the proof of Theorem 7.22 starting with “We want to show...” on line 3 from below on page 51 by the following:

We want to show that for all q , if $n + 1 + q = m + 1$ then $(n + 1, q) \in \text{rng}(f)$. Note that in fact q is uniquely determined by m and n . We have $n + p = m + 1 = n + 1 + q$, so $p = q + 1$ by Proposition 6.15(ii). Choose a such that $f(a) = (n, p)$. So $f(a) = (n, q + 1)$. Hence by definition $f(a + 1) = (n + 1, q)$, as desired. This finishes our little induction on n , and hence it also finishes our big induction on m , proving that f maps onto $\omega \times \omega$.

It now follows that $|\omega \times \omega| \leq \omega$ by Corollary 7.16.

For each $m \in \omega$ let $g(m) = (m, 0)$. Clearly g is an injection of ω into $\omega \times \omega$. Hence $\omega \leq |\omega \times \omega|$ by Corollary 7.16. Together with the preceding paragraph this gives $\omega = |\omega \times \omega|$. □

The above function f is one-one and hence is a bijection from ω onto $\omega \times \omega$. The proof that it is one-one is more complicated than the above argument, however.