

Appendix A: the axioms

These are the axioms, which were introduced at various places in the notes. Some of them involve defined notions.

Axiom 0. *There is at least one set.*

Axiom 1. (*Extensionality*) *If a and b have the same members, then $a = b$.*

Axiom 2. (*Pairing*) *For any sets a, b there is a set c which has exactly a and b as members.*

Axiom 3. (*Union*) *For any set A there is a set B such that for all x , $x \in B$ iff $x \in a$ for some $a \in A$.*

Axiom 4. (*Comprehension*) *If A is a given set and $P(x)$ is a property of sets x , then there is a set B whose elements are exactly those members x of A for which the property $P(x)$ holds.*

Axiom 5. (*Power set*) *For any set A , there is a set B whose members are exactly all of the subsets of A .*

Axiom 6. *Axiom of choice.* *For any sets A, B and any function f mapping A onto B there is a function $g : B \rightarrow A$ such that $f \circ g = \text{Id}_B$.*

Axiom 7. (*Foundation*) *If A is a nonempty set, then A has an element a such that $a \cap A = \emptyset$.*

Axiom 8. (*Infinity*) *There is a set A such that $\emptyset \in A$, and for every $a \in A$, also $a \cup \{a\} \in A$.*

Axiom 9. (*Replacement*) *If F is a class function and A is a set, then the collection of all sets $F(a)$ with $a \in A$ and a in the domain of F is a set.*