Solved problems in Monk [14]

7: Santos
8: The following were proved by Kurilic [17]:
If $\min\{\mathfrak{a},\mathfrak{b}\}\le\omega_1$, then $\mathfrak{a}(A\oplus B)=\min\{\mathfrak{a}(A),\mathfrak{a}(B)\}$.
If A is atomic, then $\mathfrak{a}(A\oplus A)=\mathfrak{a}(A)$.
37 part: Santos
38: Santos
44 part: Santos
45: Santos
46: Santos
48: Santos
49: Malliaris, Shelah
52: Santos
70: Santos
71: Santos
86: Santos
90 part: Kunen
126 part: canguzmill
149: solved by the following result, which shows that $\mathrm{hd}^{\mathrm{id}}_{\mathrm{m}}$ is trivial:

Proposition. $\mathrm{hd}^{\mathrm{id}}_{\mathrm{mm}}(A)=\omega$ *for any infinite BA A.*

Proof. Let $\langle b_n : n\in\omega\rangle$ be a partition in $\overline{A}$. Define

$$I_n=\{a\in A:\forall m<n[a\cdot b_m=0]\}.$$

Clearly $I_n\supseteq I_p$ if $n<p$. Suppose that $a\in\bigcap_{n\in\omega}I_n$ and $a\ne 0$. Choose $n\in\omega$ so that $a\cdot b_n\ne 0$, and choose $c\in A^+$ so that $c\le a\cdot b_n$. Then $c\in I_{n+1}$, so $c\cdot b_n=0$, contradiction. □

156: campcanhrumir
157: campcanhrumir
158: campcanhrumir
160: negative solution, at least for atomless BAs. Namely, we claim that $\text{h-cof}_{\mathrm{mm}}(A)=\omega$ for any atomless BA A. For, let $\langle a_i : i\in\omega\rangle$ be a system of pairwise disjoint nonzero elements of A, and for each $i\in\omega$ let $\langle b^i_j : j\in\omega\rangle$ be a system of pairwise disjoint nonzero elements less than a_i. We consider the following sequence

$$-a_0,-a_1,-a_2,\ldots(\text{rank }0)$$
$$-a_1+b^1_0,-a_2+b^2_0,-a_3+b^3_0\ldots(\text{rank }1)$$
$$-a_2+b^2_0+b^2_1,-a_3+b^3_0+b^3_1,-a_4+b^4_0+b^4_1\ldots(\text{rank }2)$$
$$\ldots\ldots$$

This gives an h-cof sequence of length ω^2. Suppose that c is adjoined at the end. Then c has rank ω, so it includes cofinally many sets $-a_n$. Since $-a_n+-a_m=1$ for $n\ne m$, it follows that $c=1$, contradiction.

References

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