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Remarks on the problems in the books Cylindric Algebras, Part I and Part II and Cylindric Set Algebras

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Part I

Part I, p. 78. The observations following Theorem 0.2.38 were not reconstructed by the authors. But G. Tardos, in 1986, did prove that there is a finitely generated pseudosimple algebra which is not simple; of course it has infinitely many operations.

Part I, p. 158, Problem 0.6: In this problem one should assume that α is less than the first uncountable measurable cardinal; then the consistency of a positive solution relative to the consistency of certain other axioms has been shown by Magidor and Laver (Trans. Amer. Math. Soc. 249, p. 97, unpublished respectively)

Part I, p. 158, Problem 0.7: For some partial results due to Németi concerning Problem 0.7 see Markusz, *On first order many-sorted logic*, Hungar. Acad. Sci. 1983.

Part I, p. 245, Problem 1.2: This problem has been solved affirmatively by Sobociński (Notre Dame J. Formal Logic 13, p. 529).

Part I, p. 246, Problem 1.4: solved negatively by Richard Thompson in 1987.

Part I, p. 263, Problem 2.3: solved affirmatively by Németi (Math. Logic in computer sci., Colloq. Math. Soc. J. Bolyai 26, p. 561).

Part I, p. 263, Problem 2.4: solved affirmatively by Ketonen for BA's, and hence for discrete CA's (Ann. Math. 108, p. 41). Using this solution, Andréka and Németi in January, 1985, extended the solution to non-discrete CA's.

Part I, p. 263, Problem 2.7: solved negatively by I. Németi.

Part I, p. 463, Problem 2.8: solved affirmatively by D. Myers (Trans. Amer. Math. Soc. 216, p. 189).

Part I, p. 464, Problem 2.9: solved negatively by Hanf (Bull. Amer. Math. Soc. 8, p. 587).

Part I, p. 464, Problem 2.10: solved by Andréka and Németi affirmatively (December, 1984).

Part I, p. 464, Problem 2.11: solved negatively ($\alpha \geq 2$) by Andréka and Németi (Notre Dame J. Formal Logic 24, p. 399).

Part I, p. 464, Problem 2.15: solved negatively by R. Krämer, R. Maddux, Alg. Univ. 15, 86-89.

Part I, p. 465, Problem 2.16: solved negatively (both questions) by H. Andréka (reference [An90] in the "Open Problems" paper, this volume).

Part II

Part II, p. vi. For information on remark (1), see below, concerning Page 180, Problems 4.11 and 4.12.

Part II, p. vi. Concerning item (5), Andréka and Németi in September 1984 showed that if $\omega \leq \alpha < \beta$, then there is a CA_β $\mathfrak{B}$ and a CA_α $\mathfrak{A}$ such that $\mathfrak{A}$ is a generating subreduct of $\mathfrak{B}$ different from $\mathfrak{Nr}_\alpha\mathfrak{B}$.

Part II, p. 103, Problem 3.2: solved negatively by Németi (January 1985). (Springer Lecture Notes in Comp. Sci. Vol. 425, 1990, p. 49.)

Part II, p. 104, Problem 3.9: solved negatively by Andréka and Németi. (J. Symb. Logic Vol. 55, 1990, pp. 577–588.)

Part II, p. 105, Problem 3.14: solved by B. Biró and G. Serény in August, 1985.

Part II, p. 179, Problem 4.2: (see also Theorem 4.1.24) Andréka and Németi obtained the following result—for $\alpha \geq \omega$ there are $2^{|\alpha|}$ varieties of CA_α's. (Ann. Pure Appl. Logic Vol. 36, 1987, pp. 235–277.)

Part II, p. 180, Problem 4.11: solved negatively by Andréka and Németi (October, 1984). (Ann. Pure Appl. Logic Vol. 36, 1987, pp. 235–277.)

Part II, p. 180, Problem 4.12: solved by Andréka and Németi in January, 1985. For $\alpha \leq 2$, the equational theory of Mg_α's is decidable, while for

$2 < \alpha \leq \omega$ it is undecidable. M. Rubin proved in January, 1985 the related result that equational theory of Mn_ω is undecidable. (Ann. Pure Appl. Logic Vol. 36, 1987, pp. 235–277.)

Part II, p. 180, Problem 4.14: solved by Andréka and Németi (January, 1985) affirmatively for both the CA and the Gs cases. A direct algebraic proof is available for all cases except CA_3.

Part II, p. 180, Problem 4.15: solved negatively by Andréka and Németi in September, 1984: for any α with $3 \leq \alpha < \omega$ we have $\mathsf{CA}_\alpha \neq \mathsf{Eq}\mathbf{K}$, where $\mathbf{K}$ is the class of all finite CA_α's.

Part II, p. 180, Problem 4.16: Two possible positive solutions were found independently by András Simon and Yde Venema. See Simon's paper in this volume, and the references there. See also reference [V91] in the "Open Problems" paper, this volume.

Part II, p. 273, Problem 5.2: solved by Richard Thompson in 1987—the first part negatively, the second part positively.

Part II, p. 273, Problem 5.3: solved positively by Andréka in 1987.

Part II, p. 273, Problem 5.4: solved negatively by Andréka in 1987. (Reference [An90] in the "Open Problems" paper, this volume.)

Part II, p. 273, Problem 5.6: solved by Richard Thompson.

Part II, p. 273, Problem 5.7: solved positively by Andréka in May, 1987.

Part II, p. 273, Problem 5.8: solved negatively for $\alpha \geq 4$ by Andréka in June, 1987, and for $\alpha = 3$ by Andréka and Z. Tuza in July, 1987.

Cylindric Set Algebras

CSA, p. 127: Problem 2.12 of Part I is still open; Maddux withdrew his claim. (For partial results see Problem 17 in the "Open Problems" paper, this volume.)

Problem 1 was solved negatively by R. Thompson. (Springer Lecture Notes in Comp. Sci. Vol. 425 p. 277, 1990.)

Problem 3 was solved negatively by Andréka and Németi in 1984. (J. Symb. Logic Vol. 55, 1990, pp. 577–588.)

Problems 5,7,8 were solved positively by Andréka, Monk, and Németi in 1984; see Part II, 3.1.139.

Problem 6—the answer is, consistently, no: done by Andréka and Németi in 1982; see Part II, 3.1.82.

Problem 10 was solved negatively by Andréka and Németi in 1984. (J. Symb. Logic Vol. 55, 1990, pp. 577–588.)

CSA, p. 221: in Figure 5.10, all ?'s should be replaced by ='s; see Part II, 3.1.139.

CSA, p. 229: Problem 6.6(vi) was solved negatively by Andréka and Németi in 1987. (J. Symb. Logic Vol. 55, 1990, pp. 577–588.)

CSA, p. 310: Problem 2 was solved positively by Andréka and Németi in 1984.

Problem 3 was solved positively by B. Biró. See the discussion and references in Shelah's paper (which is related to this problem) in the present volume.

Problem 5 was solved positively by I. Sain in 1982. (Notre Dame J. Formal Logic Vol. 29, No. 8, 1988, pp. 332-344.)

Problems 6 and 7 were solved negatively by Németi in 1985. (Springer Lecture Notes in Comp. Sci. Vol. 425, 1990, p. 49, Thm. 7.) Problem 6 was also solved independently by Biró and Shelah, see the references in Shelah's paper in this volume.

Problem 10 was solved negatively for $\alpha < \omega$ by Andréka, Comer, and Németi in 1983. For $\alpha \geq \omega$, it was solved by Sain, 1988. For an overview see Springer Lecture Notes in Comp. Sci. Vol. 425, pp. 209–225 (Sain's paper there).

Problem 11 was solved, in part, by Andréka, Monk, and Németi. (Part II, 3.1.139.)

Problem 12—consistently, the answer to the first question is no—Németi.

Problem 13—the answer to the first question is no—Andréka, Németi 1984. (J. Symb. Logic Vol. 55, 1990, pp. 577–588.)

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