

Nonfinitizability of classes of cylindric algebras

For notation see L. Henkin and A. Tarski, Cylindric algebras, Proc. Sympos. Pure Math., vol II, Amer. Math. Soc., Providence, R. I., 1961. pp. 83–113. Theorem 1. For $3 \leq \alpha$ and $K \in \omega$ there is a nonrepresentable CA_α which can be neatly embedded in a $CA_{\alpha+K}$. Theorem 2. For $3 \leq \alpha < \omega$, RCA_α is not finitely axiomatizable. Now let L_α be the elementary language for CA_α . For f a permutation of α we define $f^+ : \text{Term } L_\alpha \rightarrow \text{Term } L_\alpha$: $f v_i = v_i$; $f(\sigma + \tau) = f\sigma + f\tau$; $f(-\sigma) = -f\sigma$; $f c_K \sigma = c_{fK} \sigma$; $f d_{K\lambda} = d_{fK, f\lambda}$. A class $K \subseteq CA_\alpha$ is finite schema axiomatizable if there finitely many equations $\sigma_0 = \tau_0, \dots, \sigma_{m-1} = \tau_{m-1}$ such that K is characterized by $\{f^+ \sigma_i = f^+ \tau_i : i < m, f \text{ a permutation of } \alpha\}$. Thus for any α , CA_α is finite schema axiomatizable. Theorem 3. For $\omega \leq \alpha$, RCA_α is not finite schema axiomatizable. The proof of the central Theorem 1 uses some easy results in graph theory. (Received November 27, 1967.)