latter with a free module on countably many generators $\{x_n\}$ extending ϕ by the identity on the common summand. The submodules M_n generated by all $x - x_k$ for $k \geq n$ induce a separated topology in which x_n converges to x; their inverse images under ϕ, along with its kernel, define a norm on its domain. If the latter admitted an ultracomplete normed extension, ϕ would be extendable to a map onto x, contrary to construction. (Received November 7, 1960.)

60T-22. Donald Monk: Representation theorems for cylindric algebras.

The notation of Henkin and Tarski, Cylindric algebras, Proc. of Symposia in Pure Mathematics, vol. 3, Lattice Theory, 1960 is used. Let α be an infinite ordinal, and let $\mathfrak{A} = \langle A, +, \cdot, -, c_\kappa, d_{\kappa\lambda}\rangle_{\kappa,\lambda < \alpha}$ be a CA_α. Theorem 1. If $\mathfrak{A}$ has a Halmos-type substitution S (i.e., if $\mathfrak{A}$ is a polyadic equality algebra except for having quantifications only on finite sets), then $\mathfrak{A}$ is representable. Theorem 2. If for all nonzero $x \in A$ and for every finite subset F of α there are distinct $\kappa, \lambda \in \alpha \sim F$ such that $x \cdot d_{\kappa\lambda} = 0$, then $\mathfrak{A}$ is representable. Corollary 1. If $\mathfrak{A}$ is simple, then $\mathfrak{A}$ is representable. Corollary 2. If for all $x \in A$, $\alpha \sim \Delta x$ is infinite, then $\mathfrak{A}$ is representable. Corollary 3. If there is a finite irreflexive subset F of $\alpha \times \alpha$ such that $\prod_{\langle\kappa,\lambda\rangle \in F} - d_{\kappa\lambda} = 0$, then $\mathfrak{A}$ is representable. Theorem 1 was independently obtained by Alfred Tarski, and subsequently generalized by him. Theorem 1 cannot be strengthened by replacing "substitution S" by "substitution S acting only on finite transformations". From Corollary 1 it follows that for no β with $2 \leqq \beta < \omega$ is it true that every CA_α can be embedded in a CA_ω. Corollary 2 answers a question of Henkin, and shows that Theorem 2 generalizes the representation theorem for DCA_α's. (Received November 7, 1960.)

60T-23. P. E. Conner and E. E. Floyd: Cobordism classes of bundles.

For a space X, consider pairs (M^n, f) of compact differentiable oriented manifolds M^n and maps $f: M^n \to X$. Two such pairs (M_i^n, f_i) are cobordant if there exists an oriented $(n+1)$-manifold V^{n+1}, with boundary the disjoint union $M_1^n - M_2^n$, and a map $F: V^{n+1} \to X$ with $F|M_i^n = f_i$. The equivalence classes form a group $\Omega_n(X)$. The definition can be extended to pairs, and one obtains a homology theory satisfying the Eilenberg-Steenrod axioms, with the exception of the dimensional axiom. For a finite simplicial complex K there is a spectral sequence for which $E^2_{p,q} = H_p(K;\Omega_q)$, Ω_q the Thom group, and whose E^∞-term is

associated with a filtration of $\Omega_*(K)$. For a classifying space presents the cobordism classes of G-bundles over oriented d n-manifolds. In case all torsion of $H_*(B_G;Z)$ consists of elem we show that $\Omega_n(B_G) \approx \sum_{p+q=n} H_p(B_G;\Omega_q)$. Corresponding to bers and Pontryagin numbers of ordinary cobordism, there a mixed Whitney numbers and Pontryagin numbers, using comb n of tangential classes of the base M^n and characteristic clas For G as above, two G-bundles are cobordant if and only if co Whitney numbers and Pontryagin numbers are all equal. (Rec 14, 1960.)

60T-24. W. S. Massey: Almost complex structures on manifolds. I.

Let M be a compact, connected, orientable, differentiabl manifold whose second Stiefel-Whitney class vanishes. Theore exists a continuous field of tangent 2-frames on M, then M adm complex structure. Theorem II. If $H^2(M,Z) = 0$ and M admits a plex structure, then there exists a continuous field of tangent 2- as a consequence, the Euler characteristic of M vanishes. It fo Theorem II that the quaternionic projective plane does not admi plex structure; this answers a question raised by Hirzebruch (A vol. 60 (1954) p. 224). The example $M = S^2 \times S^2 \times S^2 \times S^2$ shows thesis $H^2(M,Z) = 0$ is essential in Theorem II. The proof of The makes use of the facts that the group of the tangent bundle of M to the group Spin (8), and that Spin (8) has interesting outer auto (Received November 14, 1960.)

60T-25. Katsumi Nomizu: Completeness of Riemannian

Some years ago H. Ozeki proved that any differentiable m satisfying the second axiom of countability admits a complete Ri It is now shown that if every Riemannian metric which M admits then M is compact. This follows from the following result. A R metric g on M is called bounded if M is bounded with respect to d(x,y) defined naturally by g as the infimum of all rectifiable cur

$m \cdot \min_{1 \le k \le m} \langle kx \rangle$ is determined. At the same time the limiting distribution ($m \to \infty$) of the integer k for which $\langle kx \rangle \le m^{-1}$ is found. Going to higher dimensions, let $x_1, x_2, \ldots$ be independent random variables, each with a uniform distribution on $[0,1]$ and define: $N(m,\gamma,p)$ = the number of integers $k, 1 \le k \le m$, for which simultaneously $\langle kx_1 \rangle \le \gamma, \langle kx_2 \rangle \le \gamma, \ldots, \langle kx_p \rangle \le \gamma$. Then it is that for fixed $0 < \gamma < 1/2$ the distribution of $N(m,\gamma,p)$ tends to a Poisson distribution with mean λ $p \to \infty$, $m \to \infty$ such that $m(2\gamma)^p \to \lambda$. (Received May 22, 1961.)

61T-167. R. J. BUEHLER, Statistical Laboratory, Iowa State University, Ames, Iowa. proofs and generalization of an optimum-gradient theorem.

In an optimum-gradient iteration, successive values $\phi_1, \phi_2, \ldots,$ of a positive definite quadratic function $\phi = x'Ax$ satisfy $\phi_2^2 \le \phi_1\phi_3$ for any initial vector x_1, and hence $\phi_{n+2}/\phi_{n+1} \ge \phi_{n+1}/\phi_n$. It is easily shown that $x_2 = (I + \lambda A)x_1$ where $\lambda = -x_1'A^2x_1/x_1'A^3x_1$, and $x_3 = (I + \mu A)x_2$, where $x_1'Ax_3 = x_1'(I + \lambda A)Ax_1 = x_2'Ax_2$. By Schwarz's inequality, $\phi_2^2 = (x_1'Ax_3)^2 \le (x_1'Ax_1)(x_3'Ax_3) = \phi_1\phi_3$. second proof, ϕ is represented in a canonical form $\sum a_i x_i^2$, and $3(\phi_1\phi_3 - \phi_2^2)/[\phi_1(\phi_2 - \phi_3)]$ is shown equal $(\sum z_i^2 a_i)^2 \sum\sum\sum z_i^2 z_j^2 z_k^2 (a_i - a_j)^2 (a_j - a_k)^2 (a_k - a_i)^2 / [\sum\sum z_i^2 z_j^2 a_i a_j (a_i - a_j)^2]^2$ where The first proof is seen to apply equally to the Hilbert space iteration considered by Temple (Proc. Roy. Soc. London Ser. A vol. 169 (1939) pp. 476-500) with A a positive Hermitian operator. In this the result is more general than that given by Akaike (Ann. Inst. Statist. Math. Tokyo vol. 11 pp. 1-16). (Received May 22, 1961.)

61T-168. DONALD MONK, University of California, Berkeley, California. Relation algebras and cylindric algebras.

Let $\mathfrak{A}$ be a CA_α with $\alpha > 2$ (Henkin and Tarski, Cylindric algebras, Proceedings of Symposia in Pure Mathematics, vol. 2, Amer. Math. Soc., 1961, pp. 83-113). Let $R(\mathfrak{A}) = \langle R(A), +, \ldots \rangle$ where $R(A) = \{a: \Delta a \subseteq 2\}$, $x;y = c_2(c_1(d_{12} \cdot x) \cdot c_0(d_{02} \cdot y))$ for all $x,y \in R(A)$, $x^{\smile} = c_2(d_{12} \ldots (d_{02} \cdot x)))$ for all $x \in R(A)$, and $1' = d_{01}$. Under certain conditions, $R(\mathfrak{A})$ is a relation algebra in the sense of Chin and Tarski, University of California Publications in Mathematics, new series no. 9, pp. 341-384). Conversely, given a relation algebra $\mathfrak{A}$ we construct a CA_3 $C(\mathfrak{A})$, using of Lyndon. Theorem 1. If $\mathfrak{A}$ is a relation algebra, then $C(\mathfrak{A})$ is a cylindric algebra, and Also, $\mathfrak{A} \cong \mathfrak{B}$ if and only if $C(\mathfrak{A}) \cong C(\mathfrak{B})$. Theorem 2. There is a relation algebra $\mathfrak{A}$ such that for no CA_5 $\mathfrak{B}$ do we have $\mathfrak{A} \cong R(\mathfrak{B})$. (Received May 23, 1961.)

61T-169. STEFAN BERGMAN, Stanford University, Stanford, California. Bounds for functions of two complex variables in a domain with a distinguished boundary set.

Let $\mathfrak{M}$ be a domain with the "smallest maximum set" $\mathfrak{F}$ in the space of two complex variables and let $\mathfrak{f}$ be a proper part of the boundary of $\mathfrak{M}$. (See Bergman, Über eine in gewissen Bereichen

$|r(T)| - B_n(T)|$ = minimum = $m(n)$. $B_n(T) \in H_n$. Since the $B_n(T)$, n $\lim_{\nu \to \infty} B_{n_\nu}(Z) = B^{(0)}(Z)$, $Z \in \mathfrak{M}$, exists. Generalizing the inequal paper, the author shows that $|f(Z)| \le C|p(Z)|$, $C = \max_{T \in \mathfrak{F}}(|f(T)| \exp(r$ $]^{-1}$, $Z \in \mathfrak{M}$. It should be noted that instead of the family of B-har other families of B-harmonic functions regular in $\mathfrak{M}$. (Received

61T-170. PETER LAPPAN, 201 Moore Street, Princeton, New Jersey. N of unbounded normal functions.

Let D denote the open unit disk and $\mathcal{C}(f)$ denote the cluster set of the func each normal meromorphic function f(z) defined in D with $\infty \in \mathcal{C}(f)$, the such that the product $f(z) \cdot B_f(z)$ is not a normal function, and (2) a such that the sum $f(z) + g(z)$ is not a normal function. The proofs i which the zeros appear at specified non-Euclidean distances from a s tends to ∞. (Received May 29, 1961.)

61T-171. O. L. MANGASARIAN, Shell Development Company, Emeryville of Wolfe's Duality theorem for nonlinear programming.

The following modification of Wolfe's duality theorem (Rand Corporation been obtained. Theorem. If x^0 is a solution of the primal problem: M and $x \ge 0$, where $\phi(x)$ is a differentiable, scalar, convex function of th by a vector each component of which $g_1(x), \ldots, g_m(x)$ is a differential then there exists some u^0 such that (x^0, u^0) is the solution of the du $x^T \nabla[\phi(x) - u^T g(x)]$ subject to $\nabla[\phi(x) - u^T g(x)] \ge 0$ and $u \ge 0$, w $\nabla = [\partial/\partial x_1, \ldots, \partial/\partial x_n]^T$; if (x^0, u^0) is a solution of the dual proble problem and in both cases the extrema are equal. The constraints Tucker constraint qualification (Nonlinear programming, Proceeding Symposium on Mathematical Statistics and Probability, 1951, p. 483). (

61T-172. A. W. AUSTIN, University of Washington, Seattle 5, Washington

of the Pontryagin duality theorem for locally compact abelian g of commutative semigroups, making use of Pontryagin's theore semigroup. A semicharacter of G is a bounded continuous complex-valu $\chi(ab) = \chi(a)\chi(b)$ for all $a, b \in G$, and not identically zero. Let $\hat{G}$ den with pointwise multiplication and the topology of uniform conver of an identity is sufficient but not necessary for $\hat{G}$ to be a (topolog