

R. McKenzie and J. D. Monk. *Chains in Boolean algebras.*

We study three invariants of a Boolean algebra A : $\underline{depth}(A)$, the sup of cardinalities of well-ordered subsets of A ; $\underline{ordinal\ depth}(A)$, the sup of ordinals embedded in A , and $\underline{length}(A)$, the sup of cardinalities of linearly ordered subsets of A . We describe how these invariants behave with respect to products, free products, formation of subalgebras, etc. Some of the more difficult results are as follows. Theorem 1. If $depth(A) = \kappa$ and $cf(\kappa) = \omega$, then depth is attained. For $depth(A) = \kappa$ limit with $cf(\kappa) > \omega$ there are examples with non-attainment. Theorem 2. If $cf(\kappa) > \omega$, A has no chain of size $cf(\kappa)$, and B has no chain of size κ , then $A * B$ has no chain of size κ . (Similarly with “type” instead of “size”). Theorem 3 (GCH) Suppose $|A| = \lambda^+$, A has a chain of type κ , $\aleph_0 \leq \kappa \leq \mu \leq \lambda$. Then A has a subalgebra of power μ and depth κ . Here GCH is necessary, and in general the theorem fails for $\mu = \lambda^+$. Theorem 4. ordinal depth is never attained. $\underline{Ordinal\ depth}(A)$ has the form $\omega^\alpha \cdot n$, where if $n = 1$ then $cf(\alpha) > \omega$. (Received september 22, 1980)