CALCULUS 3

May 4, 2009

FINAL EXAM

YOUR NAME:

◯ **001** J. Kish .... (9am)
◯ **002** T. Dent .... (10am)
◯ **003** A. Spina .... (11am)
◯ **004** A. Spina .... (12pm)
◯ **005** D. Keyes .... (1pm)

SHOW ALL YOUR WORK

final answers without any supporting work
will receive no credit *even if they are right!*
No calculators allowed.
No cheat-sheets allowed.

Partial credit will be given for any **reasonable amount of work pointing in the right direction** towards the solution of your problem. You will not get any partial credit for memorizing formulas and not knowing how to use them, or for anything you write that is not directly related to the solution of your problem.

If your test contains **more than one solution or answer** to a problem or part of a problem, and one of them is wrong, then it will be **the wrong one** the one that is **counted** for grading!

DO NOT WRITE INSIDE THIS BOX!

problem	points	score
1	10 pts	
2	10 pts	
3	10 pts	
4	12 pts	
5	15 pts	
6	20 pts	
7	16 pts	
8	25 pts	
9	20 pts	
10	25 pts	
11	25 pts	
12	12 pts	
TOTAL	200 pts	

1. **[10 pts]** Find the equation of the plane passing through the points $P(a, 0, 0)$, $Q(0, b, 0)$, and $R(0, 0, c)$, with $a \neq 0$, $b \neq 0$, and $c \neq 0$.

2. **[10 pts]** Find an equation for the tangent plane to the $x^2 - 2y^2 - 2z^2 = 12$ at the point $(1, -1, 4)$.

3. **[10 pts]** Sketch and describe the surface given by the function $z = r^2$.

4. **[12 pts]** The equation

$$x - z = \arctan(y\,z)$$

defines z implicitly as a function of x and y. Find $\dfrac{\partial z}{\partial x}\cdot$

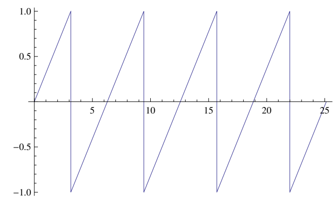
5. **[15 pts]** Find the absolute extrema of $f(x, y) = 3x^2 + 2y^2 - 4y$ on the region in the xy–plane bounded by $y = x^2$ and $y = 4$.

6. **[20 pts]** Given the vector field $\mathbf{F}(x, y) = \left(1 - ye^{-x} + e^{-y}\right)\mathbf{i} + \left(1 + e^{-x} - xe^{-y}\right)\mathbf{j}$

(a) verify that it is a *conservative field*, and

(b) find a potential function ϕ for $\mathbf{F}$.

(c) find the work performed by the force field on a particle that moves along the sawtooth curve represented by the parametric equations



$$\begin{aligned} x &= t + \arcsin(\sin t) \\ y &= (2/\pi)\arcsin(\sin t) \end{aligned} \qquad 0 \le t \le 8\pi$$

(see figure on the right).

7. **[16 pts]** Find the work done by the force $\mathbf{F}(x,y) = (1 + \tan x)\,\mathbf{i} + \left(x^2 + e^y\right)\mathbf{j}$ on the curve $\mathcal{C}$ where $\mathcal{C}$ is the boundary of the region lying between the graphs of $y = \sqrt{x}$, $y = 0$, $x = 4$, oriented counterclockwise.

8. **[25 pts]** Evaluate the surface integral $\iint_{\mathcal{S}} yz \, \mathrm{d}S$ where $\mathcal{S}$ is the surface with parametric equations

$$x = u^2 , \qquad y = u \sin v , \qquad z = u \cos v , \qquad 0 \leq u \leq 1 , \quad 0 \leq v \leq \frac{\pi}{2} \cdot$$

9. **[20 pts]** Find the *outward flux* Φ of the vector field $\mathbf{F}(x, y, z) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ through the surface of the cone $z = \sqrt{x^2 + y^2}$, $1 \leq z \leq 2$.

10. **[25 pts]** Evaluate the circulation of the vector field $\mathbf{F}(\mathbf{r}) = x^2 z\mathbf{i} + xy^2\mathbf{j} + z^2\mathbf{k}$ along the curve of intersection of the plane $x + y + z = 1$ and the cylinder $x^2 + y^2 = 9$ oriented counterclockwise as viewed from above.

11. **[25 pts]** Evaluate the *outward flux* of the vector field $\mathbf{F} = 4x^3z\mathbf{i} + 4y^3z\mathbf{j} + 3z^4\mathbf{k}$ through the surface of the sphere with radius R and center at the origin.

12. **[12 pts]** The following statements are either **true** or **false**. If true, then say so and explain why. If false, then say so and give a simple counter-example to show why the statement is false.

NOTE: Just *one lucky* example doesn't prove a statement right, but it can prove it false!

(a) The function $\dfrac{2xy}{x^2 + 2y^2}$ is continuous at the origin?

(b) If $\mathbf{F}$ is conservative, then the integral $\int_{\mathcal{C}} \mathbf{F}(\mathbf{r}) \cdot d\mathbf{r}$ is zero.

(c) div curl $\mathbf{F} = 0$.

(d) curl div $\mathbf{F} = 0$.