CALCULUS 3

April 8, 2009

3rd TEST

YOUR NAME:

◯ **001** J. KISH .... (9AM)
◯ **002** T. DENT .... (10AM)
◯ **003** A. SPINA .... (11AM)
◯ **004** A. SPINA .... (12PM)
◯ **005** D. KEYES .... (1PM)

SHOW ALL YOUR WORK

final answers without any supporting work
will receive no credit *even if they are right!*

No calculators allowed.
No cheat-sheets allowed.

Partial credit will be given for any **reasonable amount of work pointing in the right direction** towards the solution of your problem. You will not get any partial credit for memorizing formulas and not knowing how to use them, or for anything you write that is not directly related to the solution of your problem.

If your tests contains **more than one solution or answer** to a problem or part of a problem, and one of them is wrong, then it will be **the wrong one** the one that **counts** for your grading!

DO NOT WRITE INSIDE THIS BOX!

problem	points	score
1	16 pts	
2	12 pts	
3	11 pts	
4	11 pts	
5	17 pts	
6	17 pts	
7	16 pts	
TOTAL	100 pts	

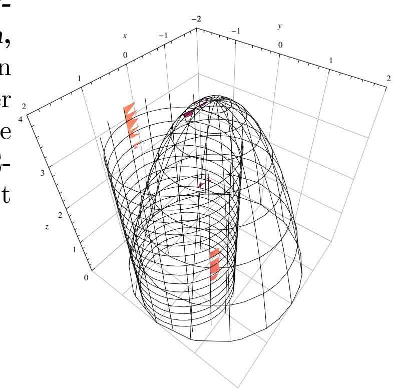
1. **[16 pts]** Find the minimum distance between the point $P_0(1, 2, 0)$ and the quadric cone $z^2 = x^2 + y^2$.

2. **[12 pts]** *Evaluate* the integral

$$\int_{y=0}^{1} \int_{x=\sqrt{y}}^{1} \sqrt{x^3+1}\ \mathrm{d}x\ \mathrm{d}y\,.$$

3. **[11 pts] Set-up, *in a coordinate system other than the rectangular and appropriate to the geometry of the problem,*** the integral that gives the surface area of the portion of the hemisphere $x^2+y^2+z^2=a^2$, $z \geq 0$ cut by the planes $z=h_1$ and $z=h_2$ $(0 \leq h_1 \leq h_2 \leq a)$. YOU DO NOT HAVE TO EVALUATE THE INTEGRAL, that is, you do not have to actually find the surface area–we just want an integral that *represents* the surface area.

4. **[11 pts] Set-up, *in a coordinate system other than the rectangular and appropriate to the geometry of the problem,*** the integral that gives the volume of the solid that is bounded on top by the paraboloid $z=4-x^2-y^2$, laterally by the cylinder $x^2+y^2=2x$ and on bottom by the xy–plane, as shown in the figure on the side. YOU DO NOT HAVE TO EVALUATE THE INTEGRAL, that is you do not have to actually find the volume–we just want an integral that *represents* the volume.



5. **[17 pts]** Evaluate

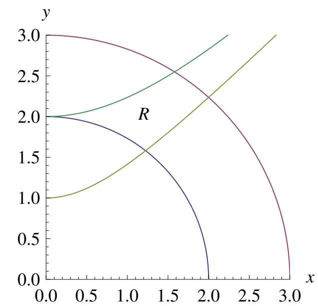
$$I = \iiint_{\mathcal{G}} \sqrt{(x^2 + y^2 + z^2)}\, \mathrm{d}V$$

where $\mathcal{G}$ is the solid between the spheres $x^2+y^2+z^2 = 1$ and $x^2+y^2+z^2 = 4$, and inside $z = \sqrt{x^2 + y^2}$.

6. **[17 pts]** Evaluate the integral

$$\iint_{\mathcal{R}} 8xy \, \mathrm{d}A_{xy}$$

over the region $\mathcal{R}$ in the first quadrant enclosed by the curves $x^2 + y^2 = 4$, $x^2 + y^2 = 9$, $y^2 - x^2 = 1$, and $y^2 - x^2 = 4$.



7. **[16 pts]** The following statements are either **true** or **false**. If true, then say so and explain why. If false, then say so and give a simple counter-example to show why the statement is false.

   NOTE: Just *one lucky* example doesn't prove a statement right, but it can prove it false!

   (a) $\displaystyle\int_{x=0}^{1}\int_{y=x^2}^{x} f(x,y)\,\mathrm{d}y\,\mathrm{d}x = \int_{y=0}^{1}\int_{x=\sqrt{y}}^{y} f(x,y)\,\mathrm{d}x\,\mathrm{d}y;$

   (b) Let $z = f(x,y)$. Then, from the surface area formula

$$S(\sigma) = \iint\limits_{\mathcal{R}} \left\| \frac{\partial \mathbf{r}}{\partial u} \times \frac{\partial \mathbf{r}}{\partial v} \right\| \mathrm{d}A_{uv}\,,$$

   where σ is parametrized by $\mathbf{r}(u,v)$, $(u,v) \in \mathcal{R}$ we can derive the surface area formula

$$S(\sigma) = \iint\limits_{\mathcal{R}} \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\,\mathrm{d}A_{xy}\,.$$

(c) If f has a relative maximum at (x_0, y_0, z_0), then $f_x(x_0, y_0) = 0$.

(d) If $f(r, \theta)$ is a constant function and the area of the region $\mathcal{S}$ is twice that of region $\mathcal{R}$, then

$$\iint\limits_{\mathcal{S}} f(r,\theta) r \,\mathrm{d}r \,\mathrm{d}\theta = 2 \iint\limits_{\mathcal{R}} f(r,\theta) r \,\mathrm{d}r \,\mathrm{d}\theta \,.$$