

CALCULUS 3

March 4, 2009

2nd TEST

YOUR NAME:

- | | |
|---|---|
| <input type="radio"/> 001 J. KISH (9AM)
<input type="radio"/> 002 T. DENT (10AM)
<input type="radio"/> 003 A. SPINA (11AM) | <input type="radio"/> 004 A. SPINA (12PM)
<input type="radio"/> 005 D. KEYES (1PM) |
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SHOW ALL YOUR WORK

final answers without any supporting work
will receive no credit *even if they are right!*

No calculators allowed.
No cheat-sheets allowed.

Partial credit will be given for any **reasonable amount of work pointing in the right direction** towards the solution of your problem. You will not get any partial credit for memorizing formulas and not knowing how to use them, or for anything you write that is not directly related to the solution of your problem.

If your tests contains **more than one solution or answer** to a problem or part of a problem, and one of them is wrong, then it will be **the wrong one** the one that **counts** for your grading!

DO NOT WRITE INSIDE THIS BOX!

problem	points	score
1	14 pts	
2	10 pts	
3	10 pts	
4	15 pts	
5	12 pts	
6	12 pts	
7	15 pts	
8	12 pts	
TOTAL	100 pts	

1. [14 pts] Given the function

$$f(x, y) = \frac{x^2 y}{x^4 + y^2}$$

find the limit $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$ or show that it does not exist.

2. [10 pts] Given the function

$$f(x, y) = \arctan \frac{y}{x}$$

find the first order partial derivatives $f_x(x, y)$ and $f_y(x, y)$.

Simplify your answers!

3. [10 pts] The radius and height of a right circular cone are measured with errors of at most 2% and 3%, respectively. Estimate the maximum percentage error in the calculated volume.

HINT: The volume of a cone of base radius r and height h is given by $V = \frac{1}{3}\pi r^2 h$.

4. [15 pts] Let w be a function of x , y , and z which is defined by $w = f(x^2 + y^2 + z^2)$.

(a) Find $w_x + w_y + w_z$.

(b) Find $w_{xx} + w_{yy} + w_{zz}$.

Simplify your answers!

5. [12 pts] The equation

$$\cos(x + y) + \cos(y + z) + \cos(x + z) = 0$$

implicitly defines z as a function of x and y . Find the partial derivative of z with respect to x .

6. [12 pts] Given the function $f(x, y) = \sqrt{x^2 + y^2}$,

- (a) Find the direction of maximum slope on the surface $z = f(x, y)$ at the point $P_0(-4, 3)$.
- (b) Find the maximum slope.

7. **[15 pts]** Find a unit vector in the direction of the tangent line to the curve of intersection of the cone $z = \sqrt{x^2 + y^2}$ and the plane $x + 2y + 2z = 20$ at the point $P_0(4, 3, 5)$.

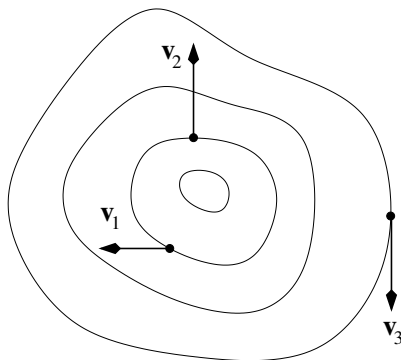
8. [12 pts] The following statements are either **true** or **false**. If true, then say so and explain why. If false, then say so and give a simple counter-example to show why the statement is false.

NOTE: Just *one lucky* example doesn't prove a statement right, but it can prove it false!

(a) If $\|\mathbf{r}(t)\| = K$ for all t , where K is a constant, then $\|\mathbf{r}'(t)\| = \text{constant}$.

(b) The curves produced in 3-space by $\mathbf{r}_1(t) = \sin t\mathbf{i} + \cos t\mathbf{j} + t^3\mathbf{k}$ and $\mathbf{r}_2(t) = \sin t^3\mathbf{i} + \cos t^3\mathbf{j} + t^9\mathbf{k}$ for $t \in [0, 1]$ are identical.

- (c) In the contour plot of the function $f(x, y)$ shown below, \mathbf{v}_1 and \mathbf{v}_3 *can not* be the gradients of f at the indicated points.



- (d) If $z = f(x, y)$ is a differentiable function of two variables, and \mathbf{u} and \mathbf{v} are two perpendicular unit vectors, then

$$\|\nabla f\|^2 = (D_{\mathbf{u}}f)^2 + (D_{\mathbf{v}}f)^2 .$$