CALCULUS 3

February 4, 2009

1st TEST

YOUR NAME:

◯ **001** J. KISH .... (9AM)
◯ **002** T. DENT .... (10AM)
◯ **003** A. SPINA .... (11AM)
◯ **004** A. SPINA .... (12PM)
◯ **005** D. KEYES .... (1PM)

SHOW ALL YOUR WORK

final answers without any supporting work
will receive no credit *even if they are right!*
No calculators allowed.
No cheat-sheets allowed.

Partial credit will be given for any **reasonable amount of work pointing in the right direction** towards the solution of your problem. You will not get any partial credit for memorizing formulas and not knowing how to use them, or for anything you write that is not directly related to the solution of your problem.

If your tests contains **more than one solution or answer** to a problem or part of a problem, and one of them is wrong, then it will be **the wrong one** the one that **counts** for your grading!

Make sure you write an arrow on top of vector quantities to differentiate them from scalar quantities (numbers). A word-processor and boldface fonts were used in writing test, but you are writing by hand! Remember that, within the same context, $\vec{v}$ (with the arrow) is a *vector* ($\vec{v} \equiv \mathbf{v}$) and v (without the arrow) is the *norm* of the previous vector ($v \equiv ||\vec{v}|| \equiv ||\mathbf{v}||$). If a vector is the null vector, write an arrow on top of the zero!

DO NOT WRITE INSIDE THIS BOX!

problem	points	score
1	10 pts	
2	10 pts	
3	13 pts	
4	13 pts	
5	13 pts	
6	09 pts	
7	10 pts	
8	10 pts	
9	12 pts	
TOTAL	100 pts	

1. **[10 pts]** Find the points P, Q, and R, at which the line $x = 1 + 2t$, $y = -1 - t$, $z = -t$ meets the three coordinate planes.

2. **[10 pts]** Determine whether the line L and plane Π are parallel or intersect? If they intersect, in how many points do they intersect?

$$L : \begin{cases} x(t) &= 1 - t \\ y(t) &= 1 + t \\ z(t) &= 1 - 3t \end{cases} \qquad \Pi : 6x - 3y - 3z = 0\,.$$

3. **[13 pts]** Determine whether the lines L_1 and L_2 are parallel, skew, or intersect.

$$\begin{aligned} L_1 :&\quad x = 4 + 2t\,, \quad y = -5 + 4t\,, \quad z = 1 - 3t\,, \\ L_2 :&\quad x = 2 + t\,, \quad\ \ y = -1 + 3t\,, \quad z = 2t\,. \end{aligned}$$

If they intersect, find the point of intersection.

4. **[13 pts]** Find the equation of the plane through the points $Q_1(1,0,0)$, $Q_2(0,0,1)$, and parallel to the line whose vector parametric equation is given by $\mathbf{r} = < t, t, t >$.

5. **[13 pts]** Consider the planes $\Pi_1 : x + z = 1$ and $\Pi_2 : y + z = 1$.

   (a) Are the planes parallel, perpendicular or neither? If neither, find the angle between them.

   (b) If your answer from part (a) was "parallel," find the distance between the two planes. Otherwise, find parametric equations for the line of intersection of the two planes.

6. **[09 pts]** In 3-space, consider the portion of the line $z = y$ (in the yz–plane) for which $z \geq 0$, and revolve it about the z-axis. Sketch the resulting surface and write its equation in spherical coordinates.

7. **[10 pts]** Write an equation for the paraboloid $z = -2x^2 - 2y^2$ in

   (a) cylindrical coordinates;

   (b) in spherical coordinates.

8. **[10 pts]** Rewrite the spherical equation $\rho = 2\cos\phi$ in rectangular coordinates.

9. **[12 pts]** The following statements are either **true** or **false**. If true, then say so and explain why. If false, then say so and explain why or give a *counter-example* to show why the statement is false.

NOTE ON THE POINTS FOR THE PROBLEM: 3 points each part, 1 point for correct *true* or *false* answer, 2 points for justification or counter-example.

(a) If $\mathbf{a} \neq \mathbf{0}$ and $\mathbf{b} \neq \mathbf{0}$, then $\dfrac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{a}\| \, \|\mathbf{b}\|} = 1$.

(b) The cross product of two unit vectors is a unit vector.

(c) If $\|\mathbf{u}+\mathbf{v}\|^2 = \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2$ then $\mathbf{u}$ and $\mathbf{v}$ are perpendicular.

(d) If $\mathbf{u}$ and $\mathbf{v}$ are any two vectors, then $|\mathbf{u} \cdot \mathbf{v}| \leq \|\mathbf{u}\| \, \|\mathbf{v}\|$.