# MATH 2400: CALCULUS 3

**March 14, 2007**

# MIDTERM 2

I have neither given nor received aid on this exam.

**Name:**________________________________________

◯ **001** E. Kim ................ (9AM)

◯ **002** E. Angel .............(10AM)

◯ **003** I. Mishev ............ (11AM)

◯ **004** J. Boisvert .......... (12AM)

◯ **005** A. Gorokhovsky ...... (1PM)

If you have a question raise your hand and remain seated. In order to receive full credit your answer must be **complete**, **legible** and **correct**. Show all of your work, and give adequate explanations.

**DO NOT WRITE IN THIS BOX!**

| Problem | Points | Score |
|---|---|---|
| **1** | 20 pts | |
| **2** | 20 pts | |
| **3** | 20 pts | |
| **4** | 20 pts | |
| **5** | 20 pts | |
| **6** | 20 pts | |
| **7** | 20 pts | |
| **8** | 20 pts | |
| **TOTAL** | 160 pts | |

**1.** Consider $\displaystyle\lim_{(x,y)\to(0,0)} \frac{2x^2y}{x^4+y^2}$.

(a) Compute the limit along the line $y = x$.

(b) Compute the limit along the parabola $y = x^2$.

(c) Based on parts (a) and (b), what is $\displaystyle\lim_{(x,y)\to(0,0)} \frac{2x^2y}{x^4+y^2}$ ?

**2.** Let $\ln(z + y - z^3) = x$. Use implicit differentiation to find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$.

**3.** Given $f(x, y) = y \sin(x)$, $P(0, 0)$, $Q(0.004, 0.003)$,

(a) Find the local linear approximation, $L$, to the function $f(x, y)$ at the point $P$.

(b) Use part (a) to approximate $f(Q)$.

**4.** A rectangular box has dimensions $x = 3$ meters, $y = 2$ meters, $z = 1$ meter. If $x$ and $y$ are increasing at 1 cm/min and 2 cm/min, respectively, while $z$ is decreasing at 2 cm/min, is the volume of the block increasing or decreasing at that instant? At what rate, is the volume changing?

**5.** Give a **unit** vector that points in the direction of maximum increase for the function $f(x, y, z) = x^3z^3 + y^3z + z - 1$ at $P(1, 1, -1)$.

**6.**

(a) Find an equation of the tangent plane to the surface $S$ given by $x^2 + y^2 + z + z^3 = 4$ at the point $P(1, 1, 1)$.

(b) Suppose that the point $Q(0.96, 1.02, z)$ is on the surface $S$ from part (a). Give the best approximation to the value of $z$ you can.

**7.** Find the absolute extrema of $f$ on $R$, where

$$f(x, y) = 2x^2 + x + y^2 - 2; \qquad R = \{(x, y) \,|\, x^2 + y^2 \leq 4\}$$

**8.** The sum of three nonnegative numbers $x$, $y$ and $z$ is 6. How large can their product be?