Calculus 3 - Theorem Summary

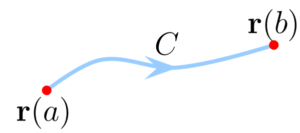
Fundamental Theorem of Calculus

$$\int_a^b F'(x)\,dx = F(b) - F(a)$$

The following theorems are all higher-dimensional versions of this one. All of them relate an integral involving derivatives over some region to the values of the original function on the boundary of that region.

Fundamental Theorem for Line Integrals

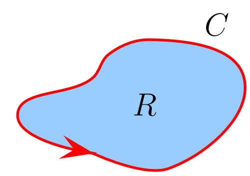
$$\int_C \nabla\phi \cdot d\mathbf{r} = \phi\big(\mathbf{r}(b)\big) - \phi\big(\mathbf{r}(a)\big)$$



Useful if the vector field $\mathbf{F}$ is **conservative**, that is, if there exists a **potential function** ϕ such that $\mathbf{F} = \nabla\phi$. Such a ϕ can be found by partial integration.

Green's Theorem

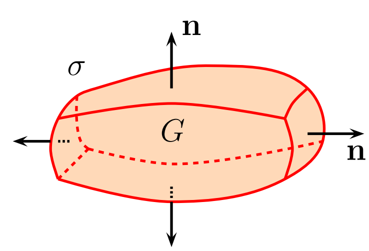
$$\oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_R \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\right) dA$$



If $\mathbf{F}$ is not conservative and C is a **closed** curve, then this theorem is very useful since the double integral may be easier to evaluate than the line integral. Green's Theorem can be used "backwards" too, for example when calculating $\text{area}(R) = \dfrac{1}{2}\displaystyle\oint_C -y\,dx + x\,dy$.

The Divergence Theorem

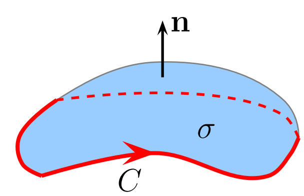
$$\iint_\sigma \mathbf{F} \cdot \mathbf{n}\,dS = \iiint_G \operatorname{div}\mathbf{F}\,dV$$



If G is a solid whose surface σ is oriented outward, then this theorem applies. If σ is a **closed** surface, the triple integral may be easier to evaluate than the flux integral.

Stokes' Theorem

$$\oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_\sigma \operatorname{curl}\mathbf{F} \cdot \mathbf{n}\,dS$$



If the closed curve C is the boundary of a piecewise smooth oriented surface σ, then $\operatorname{curl}\mathbf{F}$ can be used to find the **circulation** of $\mathbf{F}$ around C. Green's Theorem is a special case of Stokes' Theorem.