Problem 1. *Compute*

$$\oint_C \ln(1+y)\,dx - \frac{xy}{1+y}\,dy,$$

where C *is the boundary of the triangle with vertices* $(0,0)$, $(2,0)$, *and* $(0,4)$, *oriented counterclockwise.*

Answer: $\boxed{-4}$.

Problem 2. *Find the area of the region enclosed by the curve*

$$x = (1+\cos t)\cos t,\ y = (1+\cos t)\sin t,\ 0 \le t \le 2\pi$$

Answer: $\boxed{\frac{3}{2}\pi}$.

Problem 3. *Find the area of the region bounded by the curve*

$$x = \frac{3t}{1+t^3},\ y = \frac{3t^2}{1+t^3},\ 0 \le t \le 1$$

and the line $y = x$.

Answer: $\boxed{\frac{3}{4}}$.

Problem 4. *Consider the surface* S *which is a part of the plane* $3x+y-z=6$ *contained inside the cylinder* $x^2+y^2=1$ *oriented upward. Find the flux of the vector field* $\mathbf{F}(x,y,z) = xy\mathbf{i} + zy\mathbf{j} - x^2\mathbf{k}$ *across* S.

Answer: $\boxed{\iint_S \mathbf{F} \bullet \mathbf{n}\,dS = -\pi/2}$.

Problem 5. *Find the mass of a cylindrical surface of radius* $r = 3$ *centered on the* z*-axis and bounded by the planes* $z = 1$ *and* $z = 3$ *if the density function is equal to the distance to the* xy*-plane.*

Answer: The mass of the surface is $\boxed{24\pi \text{ units}}$.

Problem 6. *Compute the flux of the vector field* $\mathbf{F}(x,y,z) = x\mathbf{j}$ *out of the surface of the solid region bounded by the plane* $z = x$ *and the paraboloid* $2z = x^2+y^2$ *by*
(a) using a direct calculation;
(b) using the Divergence Theorem.

Answer: $\boxed{\iint_S \mathbf{F} \bullet \mathbf{n}\,dS = 0}$;

Problem 7. *Compute the circulation of the vector field* $\mathbf{F}(x,y,z) = x\mathbf{j}$ *along the curve* C *of intersection of the plane* $x = z$ *and the paraboloid* $2z = x^2+y^2$, *where* C *is oriented counterclockwise when viewed from above*
(a) using a direct calculation;
(b) using the Stokes' Theorem.

Answer: $\boxed{\oint_C \mathbf{F} \bullet d\mathbf{r} = \pi}$.

Problem 8. *Compute the circulation of the vector field* $\mathbf{F}(x,y,z) = y\mathbf{i} - x\mathbf{j} + xy\mathbf{k}$ *along the curve* C *of intersection of the paraboloid* $z = x^2+y^2$ *and the sphere* $x^2+y^2+z^2 = 2$, *where* C *is oriented counterclockwise when viewed from above*
(a) using a direct calculation;
(b) using the Stokes' Theorem.

Answer: $\boxed{\oint_C \mathbf{F} \bullet d\mathbf{r} = -2\pi}$.

Problem 9. *Consider the field* $\mathbf{F} = x^3\mathbf{i} + y^3\mathbf{j} + z^3\mathbf{k}$ *and the cylinder* $x^2+y^2 = 1$ *bounded by the planes* $z = 0$ *and* $z = 1$.
(a) Find the flux of $\mathbf{F}$ *out of this cylinder.*
(b) What is the flux of $\mathbf{F}$ *through just the side of the cylinder?*

Answer: $\boxed{\iint_S \mathbf{F} \bullet \mathbf{n}\, dS = \frac{5\pi}{2}}$; $\boxed{\iint_{S_{\text{side}}} \mathbf{F} \bullet \mathbf{n}\, dS = \frac{3\pi}{2}}$.

Problem 10. *Compute*

$$\int_C e^x - y^3 dx + \ x^3 + \ln y\, dy$$

where C *is the semicircle parameterized by* $\mathbf{r}(t) = 5\cos t\mathbf{i} + 5\sin t\mathbf{j}$.

Answer: $12\pi - e^2 + e^{-2}$

Problem 11. *Find the flux of the vector field* $\mathbf{F} = (xy^2 + e^{-y}\sin z)\mathbf{i} + (x^2 y + e^{-x}\cos z)\mathbf{j} + (\tan^{-1} xy)\mathbf{k}$ *across the surface of the region bounded by the paraboloid* $z = x^2 + y^2$ *and the plane* $z = 9$.

Answer: $2\pi(\frac{3^6}{4} - \frac{3^7}{7})$

Problem 12. *Find the work done by the force field* $\mathbf{F} = \langle y - x, x - z, x - y\rangle$ *in moving a particle along* C*, where* C *is the boundary of the part of the plane* $x + 2y + z = 2$ *that lies in the first octant, oriented counterclockwise as viewed from above.*

Answer: -2