Calculus 3 - Integral Summary

Line Integrals	Surface Integrals
Of functions $f(x,y,z)$:	Of functions $f(x,y,z)$:
$\int_C f(x,y,z)\,ds = \int_a^b f(\mathbf{r}(t))\,\lVert\mathbf{r}'(t)\rVert\,dt$	$\iint_\sigma f(x,y,z)\,dS = \iint_R f(\mathbf{r}(u,v))\,\lVert\mathbf{r}_u\times\mathbf{r}_v\rVert\,dA$
$= \int_a^b f(x(t),y(t),z(t))\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2+\left(\frac{dz}{dt}\right)^2}\,dt^*$	$= \iint_R f(x,y,g(x,y))\sqrt{\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2+1}\,dA^{**}$
*where C is given by $\mathbf{r}(t)=\langle x(t),y(t),z(t)\rangle$, $a\le t\le b$	**where, for instance, σ is a surface of the form $z=g(x,y)$ and R is its projection on the xy-plane. There are other, similar formulas.
Of Vector Fields $\mathbf{F}(x,y,z)=\langle f(x,y,z),g(x,y,z),h(x,y,z)\rangle$:	Of Vector Fields $\mathbf{F}(x,y,z)$:
WORK	**FLUX**
$W = \int_C \mathbf{F}\cdot\mathbf{T}\,ds$	$\Phi = \iint_\sigma \mathbf{F}\cdot\mathbf{n}\,dS$
$= \int_a^b \mathbf{F}(\mathbf{r}(t))\cdot\frac{\mathbf{r}'(t)}{\lVert\mathbf{r}'(t)\rVert}\lVert\mathbf{r}'(t)\rVert\,dt$	$= \iint_R \mathbf{F}(\mathbf{r}(u,v))\cdot\frac{\mathbf{r}_u\times\mathbf{r}_v}{\lVert\mathbf{r}_u\times\mathbf{r}_v\rVert}\lVert\mathbf{r}_u\times\mathbf{r}_v\rVert\,dA$
$= \int_a^b \mathbf{F}(\mathbf{r}(t))\cdot\mathbf{r}'(t)\,dt$	$= \iint_R \mathbf{F}(\mathbf{r}(u,v))\cdot(\mathbf{r}_u\times\mathbf{r}_v)\,dA$
$= \int_C \mathbf{F}\cdot d\mathbf{r}$	$= \iint_\sigma \mathbf{F}\cdot d\mathbf{S}$
$= \int_C f(x,y,z)\,dx+g(x,y,z)\,dy+h(x,y,z)\,dz^{***}$	$= \iint_R \mathbf{F}(x,y,g(x,y))\cdot\left\langle -\frac{\partial z}{\partial x},-\frac{\partial z}{\partial y},1\right\rangle dA^{****}$
***All of the above formulas are valid in 2 dimensions (with the necessary modifications). If the closed space curve C is the boundary of an oriented surface σ, then we get **circulation** instead of work. See Stokes' Theorem.	****where, for instance, σ is a surface of the form $z=g(x,y)$ oriented up, $G(x,y,z)=z-g(x,y)$, $\nabla G=\mathbf{r}_u\times\mathbf{r}_v$ and R is the projection of σ on the xy-plane. There are other, similar formulas.