CALCULUS 3

Math 2400

Spring 2009

Supplementary Problems

§14.1: 35, 36, 64

§14.2: 28-31, and

(i) Show that $\lim_{(x,y)\to(0,0)} \dfrac{3x^2y}{x^2+y^2} = 0$ using the definition.

§14.3: 7, 8, 11, 12, 43, 65, 74, 77, 78, 85, 86(a), 87-89, 102

§14.4:

(i) Given the function

$$f(x,y) = \begin{cases} \frac{xy}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

show that $f_x(0,0)$ and $f_y(0,0)$ both exist but f is not differentiable at $(0,0)$ [Hint: Theorem 14.4.3]. Now, find the local linear approximation of f at $(0,0)$ and explain why this is a terrible approximation.

§14.5: 16, 66, 69, 72-75, and

(i) Suppose that the equation $F(x,y,z) = 0$ implicitly defines each of the three variables x, y and z as functions of the other two: $z = f(x,y)$, $y = g(x,z)$, $x = h(y,z)$. If F is differentiable and F_x, F_y and F_z are all nonzero, show that

$$\frac{\partial z}{\partial x}\frac{\partial x}{\partial y}\frac{\partial y}{\partial z} = -1.$$

§14.6: 30-32, 61-66, 71, 75-76, 82-85, and

(i) Given $f(x,y) = \sqrt{|xy|}$ find $f_x(0,0)$ and $f_y(0,0)$. Show that these are the only directional derivatives of f that exist at $(0,0)$. Is f differentiable at $(0,0)$?

(ii) Let

$$f(x,y) = \begin{cases} \frac{x^2y}{x^4+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

First show that f has a directional derivative at $(0,0)$ in *any* direction $\mathbf{u} = \langle u_1, u_2 \rangle$. Next show that f still fails to be differentiable at $(0,0)$.

§14.7: 14, 32

§14.8: 23, 25, 26, and

(i) Look up a proof of the Second Partials Test (Theorem 14.8.6) and make sure you understand it (Stewart's Calculus book has one).

§15.1: 19-20, 30

§15.2: 41, 49, 53, 55, 60, and

(i) A useful property of double integrals is this: if $m \leq f(x,y) \leq M$ for all (x,y) in R, then

$$m \cdot \text{Area}(R) \leq \iint_R f(x,y)\, dA \leq M \cdot \text{Area}(R).$$

Use this property to estimate the following integrals:

(*a*) $\displaystyle\iint_R e^{\sin x \cos y}\, dA$, where R is the disk with center the origin and radius 2.

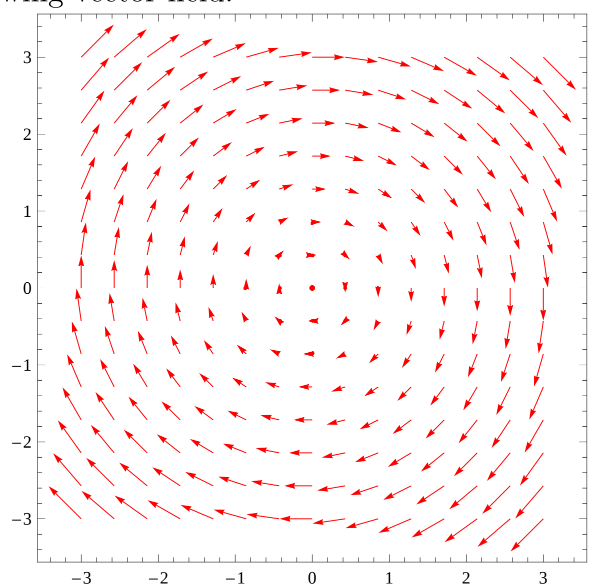
(*b*) $\displaystyle\iint_R \sqrt{x^3 + y^3}\, dA$ where $R = [0,1] \times [0,1]$.

§15.4: 13, 15, 40, 41, 42 [Hint: Try the Squeeze Theorem]
§15.5: 15, 25
§15.7: 9, 10, 14, 15, 35
§15.8: 19, 27, 31, 35, 46, 47
§16.1: 3-7, 39, 40, 43, 45, and

(*i*) Plot the gradient vector field of $f(x, y) = \sin x + \sin y$ together with a contour map of f. Explain how they are related to each other. [HINT: You may want to use a computer if you know how. You can find the answer on the last page of this document.]

§16.2: 1, 3-5, 25, 45, 47, and

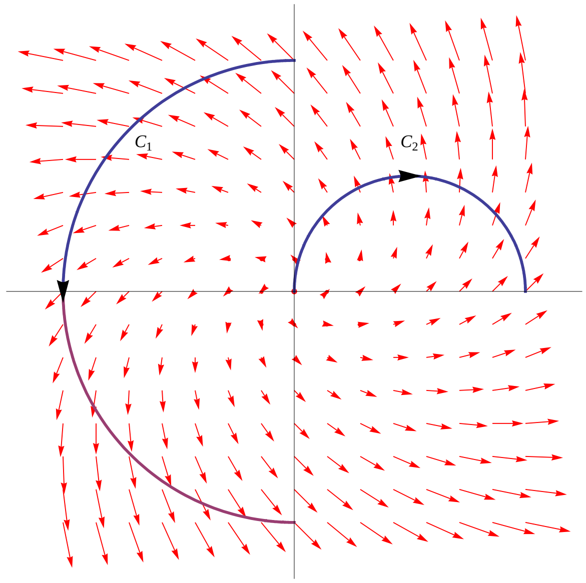
(*i*) Let **F** be the following vector field:



(*a*) If C_1 is the vertical line segment from $(-3, -3)$ to $(-3, 3)$, determine whether $\int_{C_1} \mathbf{F} \cdot d\mathbf{r}$ is positive, negative, or zero.

(*b*) If C_2 is the counterclockwise-oriented circle with radius 3 and center the origin, determine whether $\int_{C_2} \mathbf{F} \cdot d\mathbf{r}$ is positive, negative or zero.

(*ii*) The figure below shows a vector field **F** and two curves C_1 and C_2. Are the line integrals of **F** over C_1 and C_2 positive, negative, or zero? Explain.

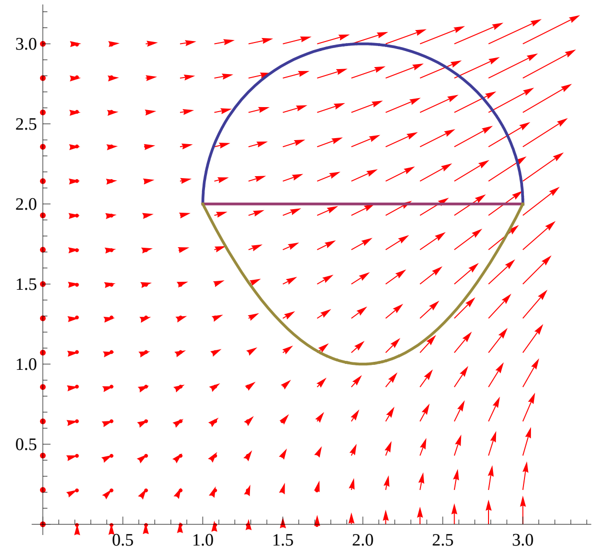


§16.3: 19, 23, 24, 32, 34, and

(*i*) The following figure shows the vector field $\mathbf{F} = \langle 2xy, x^2 \rangle$ and three curves that start at $(1, 2)$ and end at $(3, 2)$.

(*a*) Explain why $\int_C \mathbf{F} \cdot d\mathbf{r}$ has the same value for all three curves.

(*b*) What is this common value?



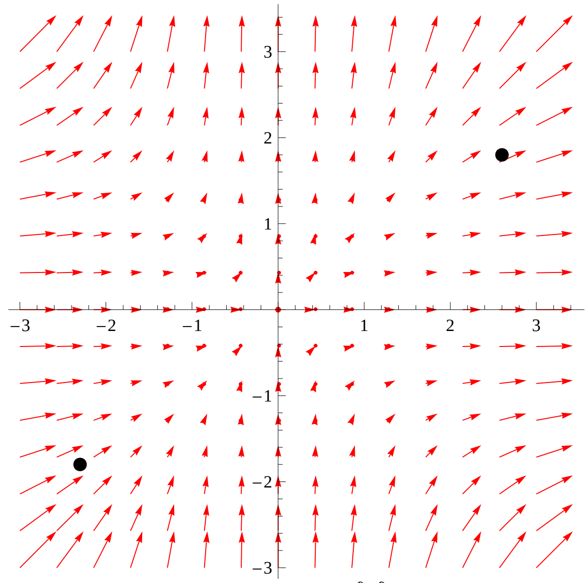
§16.4: 8, 11, 16, 22, 27, 33

§16.5: 15, 31

§16.6: 1, 4, 5

§16.7: 18, 19, 20, and

(*i*) The following graph is the vector field $\mathbf{F} = \langle x^2, y^2 \rangle$ along with two points. Are the points sinks, sources or neither? Why?



(*ii*) Use the Divergence Theorem to evaluate $\displaystyle\iint_{\sigma} (2x + 2y + z^2)\, dS$ where σ is the unit sphere $x^2 + y^2 + z^2 = 1$.

§16.8: 15, and

(*i*) Suppose that σ_1 is the upper hemisphere of $x^2 + y^2 + z^2 = 1$ for $0 \le z \le 1$ and σ_2 is the portion of $z = 1 - x^2 - y^2$ for $0 \le z \le 1$. If $\mathbf{F}$ is a vector field on $\mathbb{R}^3$ whose components have continuous partial derivatives, explain why

$$\iint_{\sigma_1} \text{curl } \mathbf{F} \cdot d\mathbf{S} = \iint_{\sigma_2} \text{curl } \mathbf{F} \cdot d\mathbf{S}$$

Answer for §16.1(i):