MATH 2400

Final Exam Review, Part 2
Fall 2008

This is a review for the chapter 16 of the final. Like part 1 of the review, you should know how to do these problems, but note that this is by no means a "complete list" of what you should know.

(1) Let $\mathbb{F}$ be a conservative vector field. Show that curl $\mathbb{F} = 0$

(2) Evaluate the following line integrals.

(a) $\displaystyle\int_C x dx + y^2 dy,$
where C is the curve $y = x^2$, $-1 \leq x \leq 2$ oriented in the positive x direction

(b) $\displaystyle\int_C \left(x + y^2\right) ds,$
where C is the curve in part (a).

(c) $\displaystyle\int_C \left(x^2 + xy\right) dx + (xy^2 - x)dy,$
where C is the counterclockwise-oriented unit circle.

(d) $\displaystyle\int_C (2xy + 1)dx + (x^2 + 2y)dy,$
where C is parameterized by $x = \frac{t^2}{\pi}\tan^{-1}(t)$, $y = \sin\left(\frac{\pi t^2}{t+1}\right)\rangle$, $0 \leq t \leq 1$

(e) $\displaystyle\int_C (z\mathbf{i} + y\mathbf{j} + x\mathbf{k}) \bullet d\mathbf{r},$
C is the intersection of the surfaces $z = \sqrt{\frac{1}{2}x^2 + y^2}$ and $x^2 + 2z^2 = 2$, oriented counterclockwise when viewed from above.

(3) Find the surface area of the part of the cylinder $x^2 + y^2 = 1$ that is between the planes $2x + 3y - z + 5 = 0$ and $z = 0$.

(4) Let R be a bound region in 2-space with boundary the clockwise-oriented curve C. Show that the area of R is given by $\oint_C y\,dx$.

(5) Evaluate the surface integrals.

(a) $\iint_\sigma x^2 y\ ds$, where σ is the part of the cylinder $x^2 + z^2 = 1$, $0 \le y \le 3$

(b) $\iint_\sigma xyz\ ds$, where σ is the part of the surface $x + 2y + 3z = 6$ in the first octant

(c) $\iint_\sigma \frac{z^2}{x^2 + y^2 + z^2}\ ds$, where σ is the unit sphere

(6) For each, find the flux $\iint_\sigma \mathbb{F} \bullet \mathbf{n}\ ds$ for the pair.

(a) $\mathbb{F} = x^3\mathbf{i} + y^3\mathbf{j} + z^3\mathbf{k}$,
σ is the outward-oriented sphere $x^2 + y^2 + z^2 = 4$

(b) $\mathbb{F} = y\mathbf{i} + x\mathbf{j} + z\mathbf{k}$,
σ is the part of the upward-oriented cone $z^2 = x^2 + y^2$ between the planes $z = 2$ and $z = 1$

(c) $\mathbb{F}$ is the curl of the vector field $2x^2yz\mathbf{i} - xy^2z^2\mathbf{j}$,
σ is the upward-oriented surface $x^2 + y^2 + z^2 = 8$ above the plane $z = 2$

(d) $\mathbb{F} = x\mathbf{i} + x^3z\mathbf{j} + (z^2 + y^2)\mathbf{k}$,
σ is the outward-oriented cube bound by $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, and $z = 1$, minus the bottom.