Math 6360 Functions of a complex variable II **Spring 2008**

General Info

Instructor: Professor David Grant, grant@colorado.edu

Office Hours: M 1-1:50, W 12-12:50, F 3-3:50 (or by appointment), in Math 303 (x2–7208).

Class Meetings: MWF 2–2:50 PM in ECCR 137.

Text: E. Freitag & R. Busum, *Complex Analysis* (Springer-Verlag)

Prerequisites. MATH 6350 or instructor's consent.

About the course.

Our focus in this course will be complex function theory, and the two main topics we will cover are the analytic theory of elliptic functions and modular functions.

Although both types of functions are used extensively in number theory, number theory will not be overly emphasized in this course. Indeed modular forms appear throughout much of mathematics, and even physics these days.

Elliptic functions are multiply periodic meromorphic functions, the generalizations of trigonometric functions to the complex plane. The periods of these functions form a 2-dimensional real lattice $L = \mathbb{Z}u + \mathbb{Z}v$, where $\tau = u/v \notin \mathbb{R}$, and switching the role of u and v if necessary we can assume τ is in the upper-half complex plane. The value of τ is not unique: all others are of the form $\gamma(\tau) = (a\tau + b)/(c\tau + d)$ for a matrix $\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ in $\mathrm{SL}_2(\mathbb{Z})$.

Modular forms are holomorphic functions $f(\tau)$ of the upper-half complex plane that transform nicely under such transformations. Namely, the differential $f(z)(dz)^k$ is invariant under this action of $\mathrm{SL}_2(\mathbb{Z})$ for some integer k. Modular forms are important because they are ubiquitous and rare (the forms for given k form a finite-dimensional complex vector space). Hence wherever you find one, you've seen it before, and that leads to powerful applications.

We will also talk about theta functions, which are mild generalizations of modular forms, which also relate to elliptic functions, which will bring us full circle.

Course requirements and grading.

I will assign homework problems from the book to individuals, and we will meet together on Wednesdays in Math 350 from 5:15 to 6:15 p.m., starting January 23 (except we will not meet on February 6, March 5 or April 9) to have you present solutions to the class. It is your responsibility to make sure the solution is correct. We will also have two in-depth written homework assignments, one on elliptic functions and one on modular forms. These presentations and your performance on the written homeworks will be the basis for your grase in this course.

Et Cetera:

The last day to drop a course without fee or a "W" on your transcript is Jan. 30. Also note that the last day for students to drop a course without petitioning the dean is Feb. 27.

Please inform me as soon as possible should you need, due to your observance of a religious holiday, to miss a homework session or a class. Provided you notify me well in advance, every effort will be made to reach a reasonable accommodation.

If you qualify for accommodations because of a disability, please submit to me a letter from Disability Services in a timely manner so that your needs may be addressed. See www.Colorado.EDU/disabilityservices.

The University of Colorado at Boulder policy on Discrimination and Harassment, the University of Colorado policy on Sexual Harassment and the University of Colorado policy on Amorous Relationships apply to all students, staff and faculty. See

http://www.colorado.edu/odh.

Further reading and resources

"Modular Functions and Dirichlet Series in Number Theory" by Apostol covers most of the material of the course. So does "Elliptic Functions" by Serge Lang.