Math 6170 Algebraic Geometry **Fall 2007**

General Info

Instructor: Professor David Grant, grant@boulder.colorado.edu

Office Hours: M 4-4:50, W 3-3:50, F 2-2:50 (or by appointment), in Math 303 (x2–7208).

Class Meetings: MWF 1–1:50 PM in ECCR 1B08.

Text: I. Shafarevich, *Basic Algebraic Geometry 1: Varieties in Projective Space* (Springer-Verlag)

Prerequisites. Two semesters of graduate algebra.

About the course.

At its core, basic algebraic geometry is concerned with studying the solutions to polynomial equations over fields k. When k is the complex numbers, these solutions form complex analytic spaces, and analytic tools like power series and differentials are available to study these spaces. The goal of algebraic geometry is to recast these tools in algebraic terms so they can be exploited for any k.

For example, when k is the field of rational numbers (or more generally, a number field), the search for solutions to polynomial equations is called Diophantine Geometry, and is a central topic in number theory (the most famous example being that of Fermat's Last Theorem, which for a prime $p \geq 3$ sought to show that $x^p + y^p + z^p = 0$ has no rational solutions with $xyz \neq 0$. This was famously proved by Wiles in 1995 using tools of modern algebraic geometry.)

Even over the complex numbers, though, the tools of algebraic geometry are very powerful because zeros of polynomials form a special class of complex analytic spaces, so more can be said about them.

Once the study begins, a few things become apparent right away. The theory is much simpler when k is an algebraically closed field (which we will assume), and is more transparent when we consider solutions to homogeneous polynomials, which can be considered points in projective space. The set of all such solutions is then called a projective algebraic set. We will put a topology (the Zariski topology) on projective space, which allows us to consider irreducible projective algebraic sets, which are called projective varieties — our basic object of study. We will study global properties of projective varieties, like their dimension, and local properties at points, like singularity. Along the way we will learn about divisors and differentials, important tools for understanding projective varieties.

Projective varieties are the objects in the catagories we study: that leaves open the question as to what the maps should be between projective varieties. There are two answers which serve different purposes: morphisms (the continuous maps), and rational maps (which are morphisms on an open set).

This introductory course covers just the basic definitions relating to projective varieties and the maps between them. The goal is to ready students for further study in the field, and to help them understand elementary applications to other fields. These applications abound and are spreading throughout mathematics and applied mathematics.

Course requirements and grading.

I will assign homework problems to individuals, and we will meet together once a week to have you present their solutions. You will also write up the solution and make copies for your classmates in case we do not have time to go over your solution at the board. It is your responsibility to make sure the solution is correct.

Et Cetera:

The last day to drop a course without fee or a "W" on your transcript is Sept. 12. Also note that the last day for A&S students to drop a course without petitioning the dean is Nov. 2.

Please inform me as soon as possible should you need, due to your observance of a religious holiday, to miss a homework session or a class. Provided you notify me well in advance, every effort will be made to reach a reasonable accommodation.

If you qualify for accommodations because of a disability, please submit to me a letter from Disability Services in a timely manner so that your needs may be addressed. See www.Colorado.EDU/disabilityservices.

The University of Colorado at Boulder policy on Discrimination and Harassment, the University of Colorado policy on Sexual Harassment and the University of Colorado policy on Amorous Relationships apply to all students, staff and faculty. See

http://www.colorado.edu/odh.

Further reading and resources

A somewhat gentler (and less detailed) introduction to the course material is Miles Reid's "Undergraduate Algebraic Geometry." There is a series of books by Ueno, the first of which is called "Algebraic Geometry 1," which covers the course material at a similar level to our book. The material is also presented in a more accelerated way in Robin Hartshorne's, "Algebraic Geometry," which is the standard text in English for the subject. After covering varieties in the first chapter, the book moves on to scheme theory and incorporates cohomological tools.