Math 3450: Introduction to Complex Variables **Spring 2018**

General Info

Instructor: Professor David Grant, grant@colorado.edu

Office Hours: M 3-3:50, W 4-4:50, F 11-11:50 (or by appointment), in Math 303 (x2–7208).

Class Meetings: MWF 2–2:50 PM in ECCR 110.

Text: E. B. Saff and A. D. Snyder, *Fundamentals of Complex Analysis with Applications to Engineering and Science.* (Pearson, 3rd ed.)

Prerequisites.

Calculus III (Multivariable calculus). A sampling of things we will need from multivariable calculus includes partial derivatives, line integrals, Green's Theorem, and 2×2 matrices. From single and multivariable calculus we will need the notions of continuity and differentiability of functions and of convergence of series and Taylor series, as well as exponential, logarithmic, and trigonometric functions and polar coordinates.

About the course.

Complex numbers are numbers of the form $p + qi$, where p and q are real numbers and i is a square root of -1 (this seems absurd, but we will make perfect sense of such expressions in a couple of different ways by expanding our notions of what a *number* is). They were invented to solve real quadratic equations (i.e., to make sense of the notion that the solutions of $ax^2 + bx + c = 0$ are

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

even when $b^2 - 4ac$ is negative.) Despite the name of the course, what we are going to study is *complex analysis*, which is essentially the 1-variable calculus of functions where the values of the variable and the function are complex numbers. Such functions which are differentiable in a region are called *analytic* functions, and are the central object of our study. Being differentiable is such a strong condition, that we can prove amazing things about analytic functions: for example, their derivatives are analytic, too. And bounded analytic functions are constants. This last fact may seem mundane, but it actually implies the Fundamental Theorem of Algebra: that every polynomial over the real (or complex) numbers has a root which is a complex number. In other words, the numbers invented to solve quadratic equations actually solve every polynomial equation. One of the magical aspects of complex analysis is that its chief tools (Cauchys Integral Formula and its corollary, The Residue Theorem) allow us to compute new sums and integrals that involve only real numbers. For example,

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6},$$

and

$$\int_{-\infty}^{\infty} \frac{\cos 2x}{x^2+1} dx = \frac{\pi}{e^2},$$

are formulas that are not easy to derive without complex analysis! If one allows complex numbers into the act, you get even more remarkable formulas, like the famous one of Euler:

$$e^{\pi i} = -1.$$

In short, one of the main things you should take away from the course is that complex analysis is beautiful and powerful. Its affect on mathematics is profound, but it is also incredibly useful throughout engineering and the sciences. Although we wont have time to talk much about these applications, one of the goals of the course is to give you the mathematical background to understand those applications.

Topical outline of the course:

We will cover (in order): Chapters 1 on the Complex Numbers (their algebraic and geometric properties, and the Riemann sphere) and Chapter 2 on Analytic functions (the differentiable complex-valued functions of a complex variable), which includes a discussion of the Cauchy-Riemann Equations (satisfied by analytic functions) and harmonic functions (the real parts of analytic functions). We will continue with Chapter 3 on Elementary Functions (polynomial, rational, exponential, logarithmic, trigonometric, hyperbolic, and inverse trigonometric functions) and Chapter 4 on Complex Integration, whose crowning achievement is the Cauchy Integral Formula. We will apply Cauchy's Integral Formula in Chapters 5 and 6. The first of these concerns the power series expansions of analytic functions, and a discussion of zeros and singularities of meromorphic functions, and the latter centers on the Residue Theorem, and its powerful applications to evaluating real integrals. We will also discuss the Gamma function, a generalization of the factorial function. If time permits, well discuss the Fourier Transform. Unfortunately, due to time constraints, we will skip most of the sections in the text on applications to science and engineering.

Class meetings.

This course will meet three days a week. This course will employ aspects of *active learning*, a technique that has proven more successful for student learning than just listening to lecturing in a course. Students spend classtime working on problems, both alone and with a partner, and then sharing ideas with the class. I will therefore not lecture all the time and have you each actively working the rest of the time.

For this reason, attendance will be manditory. Also, I will not lecture on everything in the book, so you will be responsible for reading the material in the book before classtime — I will answer questions on the material and go over the main points and do some examples. (The book is excellent and very comprehensible.)

Course requirements and grading.

Homework will be assigned weekly, and will be due the following Wednesday. All the assignments will appear on the website: euclid.colorado.edu/∼grant/courses/3450/, as will the daily outline of the course.

There will be two hour exams during our regular class time and in our usual room. The first will be on Friday, February 16, and the second will be on Friday, March 23. There will be a final exam, in our regular classroom, from 1:30 p.m. till 4 p.m. on Wednesday, May 9. Your final grade in this course will be determined by your total score out of 500 possible points. These points are broken down as follows: Homeworks count for a total of 100 points, the two hour exams will each be worth 100 points, and the final exam will make up the remaining 200 points. The final will, unlike the hour exams, be cumulative, with an emphasis on the material covered after the second exam.

You are allowed (even encouraged) to work together on homework assignments, but each of you must write up the solutions separately. In good academic fashion, if you do work with someone on a problem, just be sure to say on your homework who you worked with.

Further reading

There are many standard textbooks that cover the material in this course. A somewhat more theoretical text that is often used for this course is Complex Variables and Applications by Brown and Churchill.

Et Cetera:

The last day to drop a course without fee or a "W" on your transcript is January 31. Also note that the last day to drop a class in MyCUInfo is March 23.

Campus policy regarding religious observances requires that faculty make every effort to deal reasonably and fairly with all students who, because of religious obligations, have conflicts with scheduled exams, assignments or required attendance. Please inform me as soon as possible, and well in advance, should you need, due to religious obligations, to miss an exam, homework, or class.

Students and faculty each have responsibility for maintaining an appropriate learning environment. Those who fail to adhere to such behavioral standards may be subject to discipline. Professional courtesy and sensitivity are especially important with respect to individuals and topics dealing with race, color, national origin, sex, pregnancy, age, disability, creed, religion, sexual orientation, gender identity, gender expression, veteran status, political affiliation or political philosophy. Class rosters are provided to the instructor with the student's legal name. I will gladly honor your request to address you by an alternate name or gender pronoun. Please advise me of this preference early in the semester so that I may make appropriate changes to my records. For more information, see the policies on classroom behavior (www.colorado.edu/policies/student-classroom-and-course-related-behavior) and the Student Code of Conduct (www.colorado.edu/osccr/)

If you qualify for accommodations because of a disability, please submit your accommodation letter from Disability Services to me in a timely manner so that your needs can be addressed. Disability Services determines accommodations based on documented disabilities in the academic environment. Information on requesting accommodations is located on the Disability Services website

www.colorado.edu/disabilityservices/students.

Contact Disability Services at 303-492-8671 or dsinfo@colorado.edu for further assistance. If you have a temporary medical condition or injury, see Temporary Medical Conditions under the Students tab on the Disability Services website and discuss your needs with me.

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www.colorado.edu/policies/academic-integrity-policy

and a student Honor Code (www.colorado.edu/honorcode/). Violations of the policy may include: plagiarism, cheating, fabrication, lying, bribery, threat, unauthorized access to academic materials, clicker fraud, resubmission, and aiding academic dishonesty. All incidents of academic misconduct will be reported to the Honor Code Council (honor@colorado.edu; 303-735-2273). Students who are found responsible for violating the academic integrity policy will be subject to nonacademic sanctions from the Honor Code Council as well as academic sanctions from me. Additional information regarding the academic integrity policy can be found at the Honor Code Office website above. I will expect each student to sign the pledge of the honor code on each exam.

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