MATH 2300 **Review Sheet for Midterm #3** **Spring 2006**

This review sheet is intended as a supplement to the text, your notes, and your graded papers. The lack of a particular type of problem on this review sheet does *not* mean that type of problem won't appear on the test.

1. Use a Maclaurin polynomial of degree three to estimate the given quantity.

a. $\ln(1.25)$ **b.** $(0.7)^{-1}$ **c.** e **d.** $(8.5)^{1/3}$

2. Find the Maclaurin series for the given function, and state its interval of convergence.

a. $f(x) = x^7 e^{-x}$ **b.** $f(x) = \ln\left(1 - 6x^2\right)$ **c.** $f(x) = \cos(3x)$

3. Determine whether the series is divergent, conditionally convergent, or absolutely convergent.

a. $\sum_{i=1}^{\infty} (-1)^i \left(\frac{\ln i}{i}\right)$ **b.** $\sum_{j=0}^{\infty} \frac{(-10)^j}{j!}$ **c.** $\sum_{k=1}^{\infty} (-1)\left(\frac{-k}{k+1}\right)^k$ **d.** $\sum_{m=1}^{\infty} \frac{(m+2)!}{3^m (m!)^2}$

e. $\sum_{n=1}^{\infty} \frac{(-3)^n}{-n\left(2^n + 1\right)}$ **f.** $\sum_{k=1}^{\infty} \frac{(-1)^{k+1} k!}{1 \bullet 3 \bullet 5 \bullet \cdots \bullet (2k-1)}$ **g.** $\sum_{m=0}^{\infty} (-1)^m \left(\sqrt{m+1} - \sqrt{m}\right)$

4. Find the interval of convergence for the series, then state the values of x for which convergence is conditional (if any).

a. $\sum_{k=0}^{\infty} \frac{(x-1)^k}{\ln(k+2)}$ **b.** $\sum_{m=0}^{\infty} \frac{(x-7)^m}{m^3 4^m}$ **c.** $\sum_{n=1}^{\infty} \frac{(x-7)^n}{(2n)!}$

d. $\sum_{j=0}^{\infty} (4x)^j$ **e.** $\sum_{k=0}^{\infty} (x-4)^k$ **f.** $\sum_{n=1}^{\infty} \frac{(x+10)^n}{\sqrt{n}}$

5. Find the Taylor series generated by $f(x)$ at $x = a$.

a. $f(x) = 7^x$; $a = 3$ **b.** $f(x) = \ln\left(\frac{1+x}{1-x}\right)$; $a = 0$

6. For this problem, use the fact that $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^{2n-1}}{2n-1}$ is the Maclaurin series for $f(x) = \arctan x$.

a. What is the interval of convergence for the Maclaurin series for $f(x) = \arctan x$?

b. Find a series representation for $f(x) = \frac{1}{a^2 x^2 + b^2}$, where a and b are real constants.

c. Find a series representation for $f(x) = x^{-2} \arctan\left(x^2\right)$

7. Determine whether the given pair of polar coordinates represent the same point.

a. $\left(7, \frac{\pi}{6}\right), \left(7, \frac{7\pi}{6}\right)$ **b.** $(r, \theta), (-r, \theta + \pi)$ **c.** $\left(1, \frac{\pi}{4}\right), \left(-1, -\frac{\pi}{4}\right)$ **d.** $\left(7, \frac{\pi}{3}\right), \left(-7, \frac{-2\pi}{3}\right)$

8. Change the given polar coordinates to rectangular coordinates.

a. $\left(-9,\frac{3}{2}\pi\right)$ **b.** $\left(4,-\frac{1}{3}\pi\right)$ **c.** $\left(7,\frac{5}{6}\pi\right)$ **d.** $(\pi,3)$

9. Plot the point whose polar coordinates are given.

a. $\left(2,-\frac{5}{4}\pi\right)$ **b.** $(-1,0)$ **c.** $\left(3,\frac{1}{3}\pi\right)$ **d.** $\left(-4,\frac{1}{4}\pi\right)$

10. Graph the polar equation or inequality.

a. $r=3$ **b.** $r\le 2$ **c.** $r=2\theta$ **d.** $r=\dfrac{1}{\theta}$

e. $r=5\cos 4\theta$ **f.** $r=-\frac{1}{2}-\sin\theta$ **g.** $r=-\frac{1}{2}-\sin\theta\ \ 0\le\theta\le\frac{3}{2}\pi\ ;\ r=-4$

11. For the given polar equation, write an equivalent rectangular equation.

a. $r=\dfrac{1}{7\cos\theta-8\sin\theta}$ **b.** $r^2\sin 2\theta=20$ **c.** $r^2+2r^2\sin\theta\cos\theta=361$

12. For the given rectangular equation, write an equivalent polar equation.

a. $x^2+y^2-4x=0$ **b.** $x^2-y^2=4$ **c.** $xy=1$

13. Find the polar coordinates of the point(s) of intersection for the given curves on $0\le\theta<2\pi$.

a. $r=9+7\sin\theta,\ r=9+7\cos\theta$ **b.** $r=3,\ r^2=6\cos 2\theta$

14. Find the slope of the line tangent to the polar curve at the indicated point.

a. $r=5+5\cos\theta\ ,\ \theta=\frac{\pi}{3}$ **b.** $r=-7\csc\theta\ ,\ \theta=\frac{\pi}{4}$

c. $r=\dfrac{9}{\theta}\ ,\ \theta=4\pi$ **d.** $r=6\sin\theta-2\cos\theta\ ,\ \theta=0$

15. Find the area of the region described.

a. inside one loop of $r^2=9\cos 2\theta$ **b.** the overlap of the circles $r=4$ and $r=8\sin\theta$

c. inside $r=\sqrt{3}\sin\theta$ and outside $r=1+\cos\theta$ **d.** inside the cardioid $r=a(1+\sin\theta),\ a>0$

e. between the first and second turns of the spiral $r=\frac{3}{2}\theta$

16. Find the length of the given curve.

a. $r=\dfrac{3}{1+\cos\theta}\ ,\ 0\le\theta\le\frac{\pi}{2}$ **b.** $r=7\cos^2\left(\frac{\theta}{2}\right),\ 0\le\theta\le\pi$ **c.** $r=e^{-\theta}\ ,\ 0\le\theta$

17. Consider the inequality $x^2+y^2<3x+y$.

a. Write an equivalent inequality in polar coordinates.

b. Using the polar form, find the area of the region defined by the inequality.

c. Find the length of curve which forms the boundary of the region defined by the inequality.

d. Find the slope of the line tangent to the curve in part **c** at the point $(x,y)=(2,2)$.