# CU Boulder 

Math 4140
Test 2
Section 003 (Instructor Farid Aliniaeifard)
Friday, Mar. 23, 2018, 9:00-9:50 am

NAME (print): $\qquad$
(Family)
(Given)

## SIGNATURE:

$\qquad$

## Instructions:

1. Time allowed: 50 minutes.
2. NO CALCULATORS OR OTHER AIDS
3. There are 4 questions on 4 pages. Last page is blank.
4. Questions can be solved in more than one way.

| Question | Points | Marks |
| :---: | :---: | :---: |
| 1 | 5 |  |
| 2 | 5 |  |
| 3 | 5 |  |
| 4 | 5 |  |
| Total | 20 |  |

5. You are expected to write clearly and carefully.
6. (5 points) Let $p(x)$ be an irreducible polynomial in $F[x]$. Show that $p(x)$ is separable if and only if $p^{\prime}(x) \neq 0$.
7. (5 points) If $u$ is algebraic over $F$ and $K=F(u)$ is a normal extension of $F$, show that $K$ is a splitting field over $F$ of the minimal polynomial of $u$.
8. (5 points) If $u \in K$ is algebraic over $F$ and $c \in F$, prove that $u+1$ and $c u$ are algebraic over $F$.
9. (5 points) Let $f(x)$ and $g(x)$ be irreducible polynomials in $F[x]$ of degrees $m$ and $n$, respectively, where $(m, n)=1$. Show that if $u$ is a root of $f(x)$ in some field extension of $F$, then $g(x)$ is irreducible in $F(u)[x]$.
