CU Boulder

Math 4140

Test 1

Section 003 (Instructor Farid Aliniaeifard)

Friday, Feb 23, 2018, 9:00 - 9:50 am

NAME (print):____

(Family) (Given)

SIGNATURE:____

STUDENT NUMBER:____

Instructions:

1. Time allowed: 50 minutes.
2. NO CALCULATORS OR OTHER AIDS
3. There are 5 questions on 5 pages. Last page is blank.
4. Questions can be solved in more than one way.
5. You are expected to write clearly and carefully.

Question	**Points**	**Marks**
1	5	
2	5	
3	5	
4	5	
5	5	
Total	25	

1. (5 points) If F is a field show that $F[x]$ is not a field.

2. (5 points) Use the first isomorphism theorem to show that $\mathbb{Z}_{20}/\langle 5\rangle \cong \mathbb{Z}_5$.

3. (5 points) Let p be an irreducible elemeent of a UFD R. Show that if p divides the product of two polynomials in $R[x]$, then it must divide at least one of them. Is this statement true when R is an integral domain?

4. (5 points) Let P be an ideal in a commutative ring R with identity. Show that P is a prime ideal if and only if P has the following property: Whenever A and B are ideals such that $AB \subseteq P$, then $A \subseteq P$ or $B \subseteq P$.

5. (5 points) A monic polynomial in $R[x]$ is a polynomial whose leading coefficient is 1. Show that if R is a UFD with field of quotients F, and $f(x) \in R[x]$ is a monic polynomial, then $f(x)$ is irreducible in $R[x]$ if and only if it is irreducible in $F[x]$.

The end. Have a great weekend