

I would appreciate if you follow the following instructions:

- Staple the sheets.
- Answer in order of the asked questions.
- Properly highlight the question for which you are answering.
- Clearly mention the answers with full-forms if any.
- Be clear with handwriting and solutions.
- Write your full name.

**HW4 MATH2135, ASSIGNED: FEB. 8 - DUE: FEB. 15**

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- (1) Mark each statement True or False. Justify each answer.
  - (a) A linear transformation  $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$  is completely determined by its effect on the columns of the  $n \times n$  identity matrix.
  - (b) A linear transformation  $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$  is onto  $\mathbb{R}^m$  if every  $x \in \mathbb{R}^n$  maps onto some vector in  $\mathbb{R}^m$ .
  - (c) If  $A$  is a  $3 \times 2$  matrix, then the transformation  $x \mapsto Ax$  cannot be one-to-one.
  - (d) If  $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$  is a linear transformation, then the range of  $T$  is  $\mathbb{R}^m$ .
  - (e) The columns of a  $4 \times 5$  matrix are linearly dependent.
- (2) Do the following questions from the textbook.
  - 1.7: 5, 12, 19, 20, 21, 28
  - 1.8: 3, 6, 9, 10, 19, 20, 32, 33
  - 1.9: 18, 19, 31, 32
- (3) Do you understand the following theorem.

**Theorem.** Let  $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$  be a linear transformation and let  $A$  be the standard matrix of  $A$ . Then:

- (a)  $T$  maps  $\mathbb{R}^n$  onto  $\mathbb{R}^m$  if and only if the columns of  $A$  span  $\mathbb{R}^m$ .
- (b)  $T$  is one-to-one if the columns of  $A$  are linearly independent.