HW3 MATH2135, ASSIGNED: FEB 1, 2019 - FEB 8, 2019

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(1) Mark each statement True or False. Justify each answer.
   (a) A vector b is a linear combination of the columns of a matrix A if and only if the equation $Ax = b$ has at least one solution.
   (b) The equation $Ax = b$ is consistent if the augmented matrix $[A|b]$ has a pivot position in every row.
   (c) If the equation $Ax = b$ is inconsistent, then b is not in the set spanned by the columns of A.
   (d) If the columns of an $m \times n$ matrix A span $\mathbb{R}^m$, then the equation $Ax = b$ is consistent for each $b \in \mathbb{R}^m$.
   (e) A homogeneous system is always consistent.
   (f) The homogeneous equation $Ax = 0$ has the trivial solution if and only if equation has at least one free variable.

(2) (Bonus) Find the smallest number of vectors that span $\mathbb{R}^n$. Justify your answer.

(3) Do the following exercises in the book.
   1.3: 5, 12, 17, 18, 25, 26
   1.4: 11, 21, 22, 25, 26
   1.5: 1, 2, 5, 6, 11, 12

(4) Answer the following questions by yes or no:
   (a) Do you know what it means when we say a vector $b \in \mathbb{R}^m$ is the linear combination of a set of vectors $\{v_1, \ldots, v_n\}$?
   (b) Do you completely get the idea behind the Theorem (Theorem 3.6 in my notes)?