HW2 MATH2135, ASSIGNED: JANUARY 25, 2019- DUE: FEBRUARY 1, 2019

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First read the following carefully. Your assignments are in Page 3.

Using Row Reduction to Solve a Linear System:

(1) Write the augmented matrix of the system.
(2) By row reduction algorithm, Find an echelon form of the augmented matrix, then check the number of solutions by Theorem 1.9. If it does not have solution stop.
(3) Continue to obtain the reduced echelon form.
(4) Write the equations corresponding to the reduced echelon form in Step 3.
(5) Solve equations in a way that each basic variable is expressed in terms of free variables. Then write the set of all solutions.

Example. Previously in last lecture, the row reduced form of the augmented matrix was

$$\begin{bmatrix} 1 & 0 & -2 & 3 & 0 & -24 \\ 0 & 1 & -2 & 2 & 0 & -7 \\ 0 & 0 & 0 & 0 & 1 & 4 \end{bmatrix}$$

The equation associated to the reduced echelon form are

$$\begin{array}{ccccccc} x_1 & & -2x_3 & +3x_4 & & = -24 \\ & x_2 & -2x_3 & +2x_4 & & = -7 \\ & & & & x_5 & = 4 \end{array}$$

The pivot columns are column 1, column 2 and column 5. Therefore, the basic variables are x_1, x_2, x_5, and the free variables are x_3 and x_4. We solve the equations in terms of free variables, we have

$$x_1 = -24 + 2x_3 - 3x_4$$
$$x_2 = -7 + 2x_3 - 2x_4$$
$$x_5 = 4$$

Let $x_3 = t$ and $x_4 = s$. Then

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -24 & +2t & -3s \\ -7 & +2t & -2s \\ & t & \\ & & s \\ 4 & & \end{bmatrix} = \begin{bmatrix} -24 \\ -7 \\ 0 \\ 0 \\ 4 \end{bmatrix} + t \begin{bmatrix} 2 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} -3 \\ -2 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

So the solution set is

$$\left\{\begin{bmatrix}-24\\-7\\0\\0\\4\end{bmatrix}+t\begin{bmatrix}2\\2\\1\\0\\0\end{bmatrix}+s\begin{bmatrix}-3\\-2\\0\\1\\0\end{bmatrix}: t,s\in\mathbb{R}\right\}$$

(1) Which of the following is true and which one is false.
   (a) Every elementary row operation is reversible.
   (b) A 5×6 matrix has six rows.
   (c) A consistent system has only one solution.
   (d) An inconsistent system has only one solution.
   (e) Elementary row operations on an augmented matrix never change the solution set of the associate linear system.
   (f) Two linear system are equivalent if they have the same solution set.

(2) For each of the following linear systems:
   (a) Write the coefficient and augmented matrices.
   (b) By row reduction algorithm find the echelon form of each augmented matrix.
   (c) By using the echelon form of augmented matrix, determine the existence and uniqueness of the solution set of each linear system.
   (d) Find the basic and free variables of each of the linear systems.
   (e) Find the solution set of each of the linear systems.

$$(i) \qquad \begin{array}{rrrl} 2x_1 & +8x_2 & +4x_3 & = 2 \\ 2x_1 & +5x_2 & +x_3 & = 5 \\ 4x_1 & +10x_2 & -x_3 & = 1 \end{array}$$

$$(ii) \qquad \begin{array}{rrrrl} x_1 & +4x_2 & 5x_3 & -9x_4 & = -7 \\ -x_1 & -2x_2 & -x_3 & +3x_4 & = 1 \\ -2x_1 & -3x_2 & & 3x_4 & = -1 \\ & -3x_2 & -6x_3 & +4x_4 & = 9 \end{array}$$

$$(iii) \qquad \begin{array}{rrrrl} 2x_1 & -2x_2 & +4x_3 & -x_4 & = 6 \\ x_1 & & +3x_3 & +3x_4 & = -5 \\ 3x_1 & +6x_2 & -x_3 & & = 1 \\ 4x_1 & -4x_2 & +8x_3 & -2x_4 & = 12 \end{array}$$

(3) Do you know how to find the solution set of a linear system?