CU Boulder

Math 2130

Sample-Test 1

Section 002 (Instructor Farid Aliniaeifard)

NAME (print):____

(Family) (Given)

SIGNATURE:____

STUDENT NUMBER:____

Instructions:

1. Time allowed: 50 minutes.
2. NO CALCULATORS OR OTHER AIDS
3. There are 5 questions on 5 pages. Last page is blank.
4. Questions can be solved in more than one way.
5. You are expected to write clearly and carefully. You will be graded for both content and presentation.

Question	**Points**	**Marks**
1	5	
2	5	
3	5	
4	5	
5	5	
Total	25	

1. (5 points) Let

$$\begin{array}{cccc} & +3x_2 & -x_3 & =1 \\ x_1 & -2x_2 & +6x_3 & =0 \\ 2x_1 & -x_2 & +11x_3 & =1 \end{array}$$

Is the system consistent? if so write the solution set.

Solution. The augmented matrix is

$$\begin{bmatrix} 0 & 3 & -1 & 1 \\ 1 & -2 & 6 & 0 \\ 2 & -1 & 11 & 1 \end{bmatrix}$$

An echelon form of the matrix is

$$\begin{bmatrix} 1 & -2 & 6 & 0 \\ 0 & 3 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Since it does not have a row of the form

$$[0 \ \dots \ 0 \ b], b \neq 0$$

the system is consistent.

The reduced echelon form is

$$\begin{bmatrix} 1 & 0 & 16/3 & 2/3 \\ 0 & 1 & -1/3 & 1/3 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Therefore, x_1, x_2 are basic variables and x_3 is free. So we have

$$\begin{cases} x_1 + 16/3x_3 = 2/3 \\ x_2 - 1/3x_3 = 1/3 \end{cases}$$

Let $x_3 = t$. Then

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2/3 - 16/3t \\ 1/3 + 1/3t \\ t \end{bmatrix} = \begin{bmatrix} 2/3 \\ 1/3 \\ 0 \end{bmatrix} + t \begin{bmatrix} -16/3 \\ 1/3 \\ 1 \end{bmatrix}.$$

Thus, the set of solution is

$$\left\{ \begin{bmatrix} 2/3 \\ 1/3 \\ 0 \end{bmatrix} + t \begin{bmatrix} -16/3 \\ 1/3 \\ 1 \end{bmatrix} : t \in \mathbb{R} \right\}.$$

2. (5 points)

(a) Find a basis for

$$V = span\left\{\begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}-1\\1\\-1\end{bmatrix}, \begin{bmatrix}0\\3\\2\end{bmatrix}\right\}$$

(b) Is $b = \begin{bmatrix}0\\6\\4\end{bmatrix}$ in V?

Solution. (a) Let

$$A = \begin{bmatrix}1 & -1 & 0\\2 & 1 & 3\\3 & -1 & 2\end{bmatrix}$$

An echelon form is

$$\begin{bmatrix}1 & -1 & 0\\0 & 3 & 3\\0 & 0 & 0\end{bmatrix}$$

Since the pivot positions are in the first and second column we have

$$\left\{\begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}-1\\1\\-1\end{bmatrix}\right\}$$

is a basis.

(b) $\begin{bmatrix}0\\6\\4\end{bmatrix}$ is in V, if

$$x_1\begin{bmatrix}1\\2\\3\end{bmatrix} + x_2\begin{bmatrix}-1\\1\\-1\end{bmatrix} = \begin{bmatrix}0\\6\\4\end{bmatrix}$$

when you solve the equation you will see that the system is consistent so $\begin{bmatrix}0\\6\\4\end{bmatrix} \in V$.

3. (5 points)

   (a) Show that
   $$T(x_1, x_2, x_3) = 3x_2 - x_1 + x_3$$
   is a linear transformation.

   (b) Find the standard matrix for T.

Solution. (a) We have

- $T(x_1 + y_1, x_2 + y_2, x_3 + y_3) = 3(x_2 + y_2) - (x_1 + y_1) + (x_3 + y_3)$
$$= (3x_2 - x_1 + x_3) + (3y_2 - y_1 + y_3) =$$
$$T(x_1, x_2, x_3) + T(y_1, y_2, y_3).$$

- $T(cx_1, cx_2, cx_3) = 3cx_2 - cx_1 + cx_3 =$
$$c(3x_2 - x_1 + x_3) = cT(x_1, x_2, x_3).$$

(b) $[T(e_1)|T(e_2)|T(e_3)] = \begin{bmatrix} -1 & 3 & 1 \end{bmatrix}$.

4. (5 points)

   (a) Let B be the coefficient matrix of the linear system in question 1. Find a basis for $ColB$. What is $rankB$?

   (b) Find a basis for $NulB$. What is the dimension of $NulB$.

Solution. (a) The coefficient matrix is

$$\begin{bmatrix} 0 & 3 & -1 \\ 1 & -2 & 6 \\ 2 & -1 & 11 \end{bmatrix}$$

an echelon form is

$$\begin{bmatrix} 1 & -2 & 6 \\ 0 & 3 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

so a basis is

$$\left\{ \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix} \right\}.$$

The rank is 2.

(b) We should find the solution set of

$$\begin{bmatrix} 0 & 3 & -1 \\ 1 & -2 & 6 \\ 2 & -1 & 11 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

the reduced echelon form is

$$\begin{bmatrix} 1 & 0 & 16/3 & 0 \\ 0 & 1 & -1/3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

So

$$\begin{cases} x_1 + 16/3x_3 = 0 \\ x_2 - 1/3x_3 = 0 \end{cases} \qquad \textit{let } x_3 = t$$

Then

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -16/3t \\ 1/3t \\ t \end{bmatrix} = t \begin{bmatrix} -16/3 \\ 1/3 \\ 1 \end{bmatrix}.$$

Thus

$\left\{ \begin{bmatrix} -16/3 \\ 1/3 \\ 1 \end{bmatrix} \right\}$ is a basis for $Nul\ A$ and $dim\ Nul\ A = 1$.

5. (5 points) The last question will be True or False question.

The end. Have a great weekend