MATH 1200 (SECTION E): QUESTION ON BOARD OCT. 11

Question. Let x be an integer. If $7x + 9$ is even, then x is odd.

We present a direct proof and a proof by contrapositive.

Direct Proof. Since $7x + 9$ is even, there is an integer k such that $7x + 9 = 2k$. Therefore,

$$7x = 2k - 9 \Rightarrow$$
$$x = 2k - 9 - 6x = 2k - 6x - 10 + 1 = 2(k - 3x - 5) + 1.$$

So x is an odd number.

Proof by Contrapositive. Let x be an even integer. Then there is an integer k such that $x = 2k$. Thus,

$$7x + 9 = 7(2k) + 9 = 14k + 9 = 2(7k + 4) + 1.$$

Therefore, $7x + 9$ is an odd integer.