MATH 1200 (SECTION E): HOMEWORK 2

DUE DATE: OCT. 11 AT THE BEGINNING OF LECTURE

1. Consider each of the following pairs of statements. Decide whether each is true or false. If it is true, prove it. If it is false, explain why. Let m and n be integers.

(a) i. If m is even, then mn is even.
ii. If mn is even then m is even.

(b) i. If $m - n$ is even then both m and n are even or both m and n are odd.
ii. If $5m - 2n$ is even then both m and n are even or both m and n are odd.

Answer. (a) i. True. Let m be an even number. Then $m = 2k$ for some integer k. Thus, $mn = (2k)n = 2(kn)$. Let $kn = k^{'}$. Then $mn = 2k^{'}$ and so mn is an even number.

(a) ii. False. As an counterexample, if $m = 2$ and $n = 3$, then $mn = 6$ is even.

(b) i. We have the following cases:

Case 1. m and n both are even. Then $m = 2k$ and $n = 2l$ where k and l are integers. Thus, $m-n = 2k-2l = 2(k-l)$. Let $k^{'} = k-l$. Then $m-n = 2k^{'}$ is an even number.

Case 2. m and n are both odd. Then $m = 2k + 1$ and $n = 2l + 1$ where k and l are integers. Thus, $m - n = 2k + 1 - (2l + 1) = 2(k - l) + 2 = (k - l)$. Let $k^{'} = k - l$. Then $m - n = 2k^{'}$ is an even number.

Case 3. m is odd and n is even. Then $m = 2k + 1$ and $n = 2l$ where k and l are integers. Thus, $m - n = 2k + 1 - 2l = 2(k - l) + 1$. Let $k^{'} = k - l$. Then $m - n = 2k^{'} + 1$ is an odd number.

Case 4. m is even and n is odd. Similar to Case 3, we have $m - n$ is odd.

Therefore, by the above cases the only possible situations that $m - n$ is even is when both m and n are even or both m and n are odd.

(b) ii. False. As a counterexample, assume $m = 2$ and $n = 1$. Then $5m - 2n$ is even but m is even and n is odd.

Exercise (you do not have to submit it). What is the negation of "both m and n are even or both m and n are odd."

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2. (a) Look at the following statements:

A: m and n are odd.
Write the negation of A.

(b) (Bonus) Prove the following statement by the method of contrapositive.
If mn is odd then both m and n are odd.

Answers. 2(a) $\neg A : m$ or n is even.

2(b). We proceed the proof by contrapositive. Let m or n be even. So we have two cases:
Case 1. m is even. Therefore, $m = 2k$ for some integer k. Thus, $mn = (2k)n = 2(kn)$. Let $kn = k^{'}$. Then $mn = 2k^{'}$ and so mn is an even number.
Case 2. n is even. Therefore, $n = 2k$ for some integer k. Thus, $mn = m(2k) = 2(km)$. Let $km = k^{'}$. Then $mn = 2k^{'}$ and so mn is an even number.