

MATH 1200 (SECTION E): ANSWERS TO HOMEWORK 1

DUE DATE: SEPT. 27 AT THE BEGINNING OF LECTURE

1. Determine and prove if the following statements are true or false. For false statements, change them as to make them True.

- (a) $\{2\} \subseteq \{1, \{2\}, 3\}$
- (b) $\{1\} \subset \{1, 2, 3\}$
- (c) $\{1, 2, 3\} \subset \{1, 2, 3\}$

Answers.

1 (a) False. Because $\{2\}$ is an element of $\{1, \{2\}, 2\}$ not a subset. Correct version: $\{2\} \in \{1, \{2\}, 3\}$

Review (This is not part of answer): A set A is a subset of B , denoted by $A \subseteq B$, if every element of A is an element of B .

1 (b) True. Because 1 is an element of $\{1, 2, 3\}$, therefore, every element of $\{1\}$ is an element of $\{1, 2, 3\}$. Thus, $\{1\} \subset \{1, 2, 3\}$.

1 (c) False. We write $A \subset B$, when $A \subseteq B$ but $A \neq B$. Correct version $\{1, 2, 3\} = \{1, 2, 3\}$.

2. Show that the sum of the first n odd numbers is n^2 .

There are different ways to proof this statement, we write one of them.

Answer. Let the sum of the first n odd numbers be S_n . Then

$$S_n = 1 + 3 + 5 + \dots + (2n - 1).$$

Take two copies of S_n , and add them together, so we have

$$\begin{array}{cccccccccccc} & & & & & S_n + S_n = & & & & & & & & & & & & & & & & \\ & 1 & + & 3 & + & 5 & + & \dots & + & (2n - 5) & + & (2n - 3) & + & (2n - 1) & & & & & & & \\ + & \\ & (2n - 1) & + & (2n - 3) & + & (2n - 5) & + & \dots & + & 5 & + & 3 & + & 1 & & & & & & & \\ \hline & 2n & + & 2n & + & 2n & + & \dots & + & 2n & + & 2n & + & 2n & & & & & & & & \end{array}$$

We have $2n$, n times. Therefore, $2S_n = n(2n)$, so $S_n = n^2$.