MATH 1200 (SECTION E): HINTS-SAMPLE QUESTIONS FOR CLASS TEST 1

1. Let A be the set

$$\{\alpha, \{1, \alpha\}, \{3\}, \{\{1, 3\}\}, 3\}.$$

Which of the following statements are true and which are false? Justify your answer.
(a) $\alpha \in A$. True, since α is a member of A.
(b) $\{\alpha\} \notin A$. False. Note that α is a member of A but $\{\alpha\}$ is not. Actually, $\{\alpha\} \subseteq A$.
(c) $\{1, \alpha\} \subseteq A$. False, because $\{1, \alpha\} \in A$.
(d) $\{3, \{3\}\} \subseteq A$. True, because the members of $\{3, \{3\}\}$, i.e., 3 and $\{3\}$ are members of A.
(e) $\{1, 3\} \in A$. False.
(f) $\{\{1, 3\}\} \subseteq A$. False, $\{\{1, 3\}\} \in A$
(g) $\{\{1, \alpha\}\} \subseteq A$. True, because $\{1, \alpha\}$ is a member of A.
(h) $\emptyset \in A$. False.
(i) $\emptyset \subseteq A$. True.

2. Show that $1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$.
Hint. Same technique as Question 2 of Assignment 1.

3. If A implies B which of the following?
(a) Either A is true or B is true. **False**
(b) $A \Rightarrow B$. **True**
(c) $B \Rightarrow A$. **False. This is the converse.**
(d) $\neg A \Rightarrow B$. **False**
(e) $\neg B \Rightarrow A$. **False**

4. Determine only which of the following is true or false.
(a) If $a > b$, then $3a$ is necessarily $> 2b$.
(b) The set of all x which satisfy the inequality $|x^2 - 5| > 4$ is all x such that $|x| > 3$.
(c) IF $x > y$, then necessarily $|x| > |y|$.
Hint. (a) and (b) are False, but you need a counterexample if we say disprove them.

5. Which of the following are True or False. Justify your answers.
(a) If $n^2 - 2n - 3 = 0$, then $n = 3$.
(b) For integers a and b, if ab is a square, then a and b are squares.
(c) For integers a and b, ab is a square if a and b are squares.

Hints. (a) This is False, because ... .
(b) False, find a counterexample.
(c) This is same as the following statement: "if a and b are squares, then ab is a square ". This is true. Let $a = x^2$ and $b = y^2$, then ... .

6. Disprove the following statements.
(a) If n and k are positive integers, then $n^k - n$ is always divisible by k.
(b) Every positive integer is the sum of three squares (the squares being $0, 1, 4, 9,$ etc).
Hint. Find counterexamples. For (a) let $n = 2$ and $k = 4$.

7. If n is an even integer, then $n^2 + 4n + 3$ is odd.
Hint. Since n is even, $n = 2k$ for some integer k. Then substitute n by $2k$, and finally you should show that the result is odd.

8. Let n be an integer such that n^2 is a multiple of 3. Then n is also a multiple of 3.
Hint. Look at Example 1.3 page 5 of the recommended textbook.

9. If for an integer n, $5n^2 + 2n + 3$ is even, then n is odd.
Hint. Use Contrapositive. Let n be even, then you should have $5n^2 + 2n + 3$ is odd.

10. Let n be an integer. Then n is even if and only if n^2 is even.
Hint. "If and only if" means you should prove both $A \Rightarrow B$ and $B \Rightarrow A$. So for this question you should prove:
(1) If n is even, then n^2 is even.
(2) If n^2 is even, then n is even (use contrapositive).