MATH 1200 (SECTION E):TEST 1 OCTOBER 25,
DURATION: 75 MINUTES

Full Name:
Student number:

1. Let $A = \{1, \{2\}, 3\}$ and $B = \{1, 2\}$. Which of the following are true or false. Justify your answer.
(a) $B \subseteq A$.
(b) $\{2\} \in B$.
(c) $\{1, \{2\}\} \subseteq A$.

(a) False since $2 \notin A$.

(b) False since $\{2\} \notin B$.

(c) True since both 1 and $\{2\}$ are elements of A.

2. Disprove the following statements.
(a) If $a > b$, then $3a$ is necessarily $> 2b$.
(b) The set of all x which satisfy the inequality $|x^2 - 4| > 4$ is all x such that $|x| > 3$.
(c) If $n^2 - 6n + 8 = 0$, then $n = 4$.

(a) Let $a = -3$ and $b = -4$, then $3(-3) \not> 2(-4)$.

(b) If $x = 2.9$, then $|x^2 - 4| > 4$, but $|2.9| \not> 3$.

(c) when $n = 2$, $n^2 - 6n + 8 = 0$.

3. Let n be an integer. Then $\underbrace{n \text{ is even}}_{A}$ if and only if $\underbrace{n^2 \text{ is even}}_{B}$.

$A \Rightarrow B$) Let n be an even integer. Then $n=2k$ for some integer K. Thus, $n^2=(2k)^2=4k^2=2(2k)$ is even.

$B \Leftarrow A$) (Contrapositive) Let n be odd. Then $n=2k+1$ for some integer k. Thus, $n^2=(2k+1)^2=4k^2+\cancel{2}4k+1=$
$2(2k^2+2k)+1$ is an odd integer

4. Let n be an integer. If $3n^2+4n+5$ is even, then n is odd.

(Contrapositive) Let n be an even integer.
Then $n=2k$ for some integer k. Thus,

$$3n^2+4n+5=3(2k)^2+4(2k)+5=$$
$$12k^2+8k+5=$$
$$2(6k^2+4k+2)+1$$ is an odd integer.

5. Prove the following statement by contrapositive.
Let m and n be integers. If mn is even, then m or n is even.

(contrapositive) Let both m and n be odd integers. Then ~~make~~ $m = 2k+1$ and $n = 2l+1$ for some integers k and l.

~~to~~ we have

$$mn = (2k+1)(2l+1) =$$

$$4kl + 2k + 2l + 1 =$$

$$2(2kl + k + l) + 1$$ is an odd integer.